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2005

ABSTRACT

An Empirical and Theoretical Analysis of Capital Asset Pricing Model

by

M. Sharifzadeh

M. Phil., University of Oxford, England, 1972
B.S., University of Salford, England, 1970

Dissertation Submitted in Partial Fulfillment
of the Requirements for the Degree of
Doctor of Philosophy
Applied Management and Decision Science

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ABSTRACT

The problem addressed in this dissertation research was the inability of the single-factor capital asset pricing model (CAPM) to identify relevant risk factors that investors consider in forming their return expectations for investing in individual stocks. Identifying the appropriate risk factors is important for investment decision making and is pertinent to the formation of stocks' prices in the stock market. Therefore, the purpose of this study was to examine theoretical and empirical validity of the CAPM and to develop and test a multifactor model to address and resolve the empirical shortcomings of the single-factor CAPM. To verify the empirical validity of the standard CAPM and of the multifactor model five hypotheses were developed and tested against historical monthly data for U.S. public companies. Testing the CAPM hypothesis revealed that the explanatory power of the overall stock market rate of return in explaining individual stock's expected rates of return is very weak, suggesting the existence of other risk factors. Testing of the other hypotheses verified that the implied volatility of the overall market as a systematic risk factor and the companies' size and financial leverage as nonsystematic risk factors are important in determining stock's expected returns and investors should consider these factors in their investment decisions. The findings of this research have important implications for social change. The outcome of this study can change the way individual and institutional investors as well as corporations make investment decisions and thus change the equilibrium prices in the stock market. These changes in turn could lead to significant changes in the resource allocation in the economy, in the economy's production capacity and production composition, and in the employment structure of the society.

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CHAPTER 1:

INTRODUCTION TO THE STUDY

Introduction

The pricing of assets like stocks and bonds that trade in the capital market is one of the most important areas of finance and investment and affects the economic life of both individuals and organizations. According to economic theory the value of any asset, including the value of assets trading in the capital market, depends on three components (a) the expected or future cash flows from the asset, (b) the timing of those expected cash flows, and (c) an expected or required rate of return that is used to discount all the expected future cash flows, the sum of which will be the basis of the asset's value (Cochrane, 2001, pp. 5-28). As future cash flows from assets are uncertain, and the extent of uncertainty of cash flows differs from asset to asset, investors' expected or required rates of return from assets differ across different assets or different asset classes. And this difference between required rates of return on different assets reflects varying degrees of risks that investors have to take when investing in different assets. Therefore, two assets that are similar in all aspects and the same pattern of cashflows are expected from them will trade at different prices in the market if investors assign different degrees of risk, or uncertainty of future cash flows, to them. This dependence of the expected rate of return of an asset on the risk embodied in the asset makes the expected rate of return concept and its relationship with some measures of risk the most fundamental issue, both theoretically and practically, for asset valuation. The capital asset pricing model (CAPM), which is the subject matter of this dissertation, is the most referenced theory that tries to explain the relationship between risk and expected rate of return and thus provide a conceptual method to determine the most important component of the asset valuation problem.

The CAPM, which was independently developed by Sharpe (1964), Linter (1965), and Mossin (1966), makes certain assumptions about the behaviors of the investors and about the working of the capital market and on the basis of those assumptions derives a specific linear relationship between expected rate of return and risk; a relationship that according to CAPM should hold for every individual asset or any combination of individual assets in order for the capital market to be in equilibrium. The analytical findings of CAPM as wells as major empirical tests conducted for its validity will be discussed at length in the literature review section of this dissertation. As for this introduction it is pertinent to point out that the main finding of CAPM, which is the subject of most controversies and this author intends to make his contribution on the issue, is related to the concept of rate of return versus risk in the capital market.

Early empirical tests of the CAPM such as those by Linter (1965); Black, Jenson, and Scholes (1972); and Fama and MacBeth (1973) concentrated on the linearity of the relationship between rates of return and beta for cross section of securities. Later empirical tests of the model initiated by Fama and French (1992) focused on the anomalies in the CAPM framework. These tests tried to investigate whether other variables like size and book-to-market value ratio, besides the beta, could explain the variation of average rates of return for cross-section of securities. However, although at this point in the literature these new tests have provided overwhelming evidence that more than one risk factor is at work explaining stock returns but most of the tests fail to justify that the variables they add to the model are really additional risk factors that investors want to be compensated for and, therefore, do not back their findings by a theoretical model. It is the contention of this author that the main reason for such shortcomings is that in the field of investment theory the concept of risk factor is treated in isolation from the perception of risk factor in the field of corporate finance theory.

Statement of the Problem

The question of what determines the equilibrium prices in the stock market, or what is the right price for an investor to buy or sell a stock in the stock market, leads to the question of what risk factors investors consider, or should consider, in determining their expected rates of return from the stocks. This highlights the need to have a suitable model that can identify such risk factors and explain how investors' expected rates of return are determined. The overextended stock prices in the second half of the 1990s which led to the stock market collapse of the year 2000 casts doubts on the ability of the single-factor CAPM to explain how investors' return expectations are formed and thus its inability to explain and predict stock prices. Therefore, the problem to be addressed in this study is the inability of the single-factor CAPM to identify relevant risk factors that investors consider in forming their return expectations and the relationships between those relevant risk factors and return expectations. Thus, in this study a multifactor model containing five risk factors is suggested and tested against empirical data to provide a better explanation of the factors that determine the stocks rates of return than the single-factor CAPM does.

The knowledge of what factors determine investors' return expectations enables investors to evaluate if at any particular point of time the stock or portfolio of stocks they are considering to invest in or are holding is undervalued, overvalued, or properly valued by the market and thus make the right buy, sell, or hold decision. As was mentioned in the introduction section, theory of investment posits that investors value any investment, including investment in the stocks of companies, by discounting expected or predicted future cash flows from that investment at a discount rate commensurate with the inherent risk of the investment. In the investment and finance literature, this discount rate that investors use to value a stock or portfolio, is referred to as the *expected rate of return* of the stock or portfolio. The problem of stock or portfolio valuation, therefore, leads to the following questions: (a) *how do investors form their return expectations from an investment* (b) *what is the relationship between the expected rate of return and the risk of investing in a stock or portfolio, and (c) are there other possible variables besides the risk that affect the expected rate of return of the investment*?

Background of the Problem

In today's U.S. society many people are directly or indirectly involved in the capital market. People might invest their surplus income through the stock market by themselves into individual accounts or into retirement accounts, like 401K or self-directed IRA's. Or they might put their savings into managed accounts like mutual funds or investment trusts. Institutional investors, like mutual funds, insurance companies, foundations, and charities, deal with huge amounts of money that they invest on behalf of their investors through the stock market in pursuit of specific objectives. It is now even advocated to privatize part of the Social Security Fund and let it be invested in the stock market. In all these cases people or institutions are faced with a set of issues; how much of their funds to invest through the stock market, how to allocate those funds between broad investment classes like stocks and bonds, what specific stocks or bonds to invest in, and more importantly what is the correct price for buying or selling any specific stock, bond, or a portfolio of stocks and bonds. Individual investors who entrust their savings with mutual funds and other institutional investors need to have some criteria to assess if their savings have been well invested. Institutional investors need to regularly evaluate the performances of their fund managers and revise their fund manager selections. On the other side of the spectrum, when corporations issue new securities, especially when issuing new equities to finance new investment projects, need to know at what price to offer the new securities to the capital market; and the price at which those securities can be absorbed in the capital market affects their decisions to continue or not with the new investment projects. In certain industries, specifically in the regulated utilities industry, the price to be charged to the consumer is related to the price of the company's stock or bond in the capital market, which in turn determines the cost of capital for the utility company.

All of these important issues have one common denominator, and that is the problem of how securities are or should be priced in the capital market. The CAPM developed by Sharpe (1964), Linter (1965), and Mossin (1966), which has been subject to various empirical tests since its inception, addresses this problem through relating the expected rate of return of a security to its systematic risk. This study will differ from the standard CAPM and the previous tests of the model in two aspects. First, although theoretically the standard CAPM should hold for every individual security as well as for any combination of securities, most tests of the standard CAPM concentrate on portfolios rather than individual securities. If it is empirically shown that CAPM holds for portfolios then its usefulness would be only in the domain of portfolio management, whereas if an investment theory works for individual securities it can be useful not only for portfolio management but also for individual investments, the cost of capital problem, the capital budgeting and capital structure decisions of corporations, and for all the other problems mentioned in the beginning paragraph of this introduction. Therefore, the first intention in this study would be to test the standard single-factor CAPM for individual securities over the sample period. Second, as the author's pilot study suggests, and is expected so from the test of hypotheses on single-factor CAPM, the systematic risk (beta) by itself has a very weak explanatory power and therefore there must be some other risk factors accounting for differences in average rates of return across securities. In Chapter 3 of this dissertation some specific risk factors, backed by both corporate finance theory and investment theory, will be suggested as additional risk factors and a multifactor model will be proposed and tested against empirical data.

Nature of the Study

This study is quantitative, explanatory, and uses available data to investigate correlations and examine regressions amongst variables. Therefore, the research design in this study would be casual-comparative. The reason for employing the causal-comparative design is that the independent variables of the study can not be manipulated experimentally and thus it is not possible to investigate the relationship between the dependent variable and the independent variables through experimental designs.

Purpose of the Study

The reason why this study is proposed is that in the second half of the 1990s, which was labeled as the era of *the new economy* and information communication revolution, the US stock market faced an unprecedented rise in the stock prices which was followed by the stock market crash in April of 2000. And not many tests have been done for post-1995, and in particular there seems to be no empirical study on CAPM distinctly covering and both the second half of the 1990s and post-2000 where general economic situations were different form pre-1995. Thus, this study will investigate the validity of both the standard CAPM and the multifactor model developed by the author for the whole time span of the research, January 1995 to December 2004.

Theoretical Basis of the Study

Modern investment theory started with Markowitz portfolio investment theory. This theory was developed by Harry Markowitz in 1952 and earned him the Nobel Prize in economics. Markowitz (1952) postulated that the rates of return of individual assets covary with one another, and there is a rather stable covariance, or correlation coefficient, between the rates of return of every two assets. Thus, he stated that it is theoretically possible to construct a variance-covariance matrix of all risky assets. Knowing the variance-covariance of returns for all risky assets makes it possible to mathematically compute the risk, defined as the standard deviation of returns, of any portfolio consisting of specific weights of each asset. Through a sophisticated mathematical analysis Markowitz proved that to achieve a desired or expected rate of return on a combination of risky assets there is a specific mix of those assets, or optimal weights, that carry a minimum risk. Putting it in another way, Markowitz showed that for any level of risk that an investor can tolerate there is an optimal weight of assets that yields a maximum rate of return on the portfolio. Portfolios built with optimal weights of their constituent assets were called *efficient* portfolios by Markowitz. He further showed that, if the rates of return on different efficient portfolios are plotted against their risks on a two dimensional graph, the result would be a smooth curve which he called it *the efficient frontier*. To complete his portfolio selection model, Markowitz made the assumption that all investors are *risk averse*, meaning that for a specific level of return they prefer the portfolio with the least risk or what comes to the same thing, for a given level of risk tolerance investors choose the portfolio with the maximum rate of return. This implies that every risk-averse investor will choose a portfolio on the efficient frontier that suits his or her risk-returns profile. The efforts of all investors to build their desired efficient portfolios lead them to buy or sell some securities in the capital market and the outcome of all these decisions is establishment of the equilibrium asset prices in the capital market.

Markowitz model, although is still regarded as a sound theory of investment, but at the time of its inception it was practically difficult, if at all possible, to be tested empirically or to be implemented by investment professionals. The model requires calculating the variance-covariance matrix of all risky assets which means, for example, for the stocks in the S&P 500 Index, ½*(500) (501) or 125,250 variances and covariances need to be calculated, a task that was not worthwhile before fast computers. The Markowitz model inspired other researchers to incorporate Makowitz's risk-return ideas into less complicated models. And, these efforts led to the development of the CAPM.

Theoretical foundation of capital asset pricing model was originally developed by Sharpe in 1963 and subsequently elaborated into the equilibrium model of the capital market prices by Sharpe (1964), Linter (1965), and Mossin (1966). In the 1970s after techniques for estimating the required inputs to the model were elaborated, it was packaged and marketed as computer software to the mutual funds and other institutional investors. From there, modern investment theory took off in terms of practical applications and up to the present time many institutional investors and investment professionals adhere to the predictions of CAPM in making investment decisions and managing investment portfolios. However, with present computational capacity of computers, the Markowitz model is now implemented to allocate investments between classes of securities, such as, between stocks and bonds, and the CAPM is used to allocate funds between different stocks within the equity part of the portfolio.

The basic tenet in the CAPM is that the reason why rates of return of individual stocks covary with one another is because the rate of return of every stock or any portfolio of stocks varies with a common factor, and that common factor is the rate of return of the overall stock market. The overall market is the portfolio of all risky assets, in which every asset is weighted by the dollar market value of that asset relative to total market values of all assets. For practical implementation of the CAPM findings and for test of hypotheses purposes a broad value weighted index, like the S&P 500 Stock Index, is usually taken as the proxy for the overall market. With this idea and by making several assumptions regarding investors' behaviors and the working mechanism of the stock market, the CAPM derives the followings conclusions regarding the asset pricing in the capital market:

1. The rate of return of every individual stock is a linear function of the rate of return of the overall stock market. The line representing this relationship is called the

characteristic line of the stock. The slope of this line represents the sensitivity of the rate of return of a stock to the rate of return of the overall market and is denoted by β (beta) in the literature.

2 The beta of a stock represents the systemic risk of a stock. More precisely, the systematic risk of a stock is the square of its beta times the variance of the overall market rate of return. Systematic risk is the only risk that matters to the investors. Any variation in the rate of return of a stock in excess of its systematic risk is specific to the stock and can be eliminated through diversification.

3. If the rates of return of individual stocks are plotted against their betas, the result would be an upward-sloping line, called *the security market line*. The slope of the security market line is the rate of return of the market portfolio over the risk-free rate in the economy; and is called stock market risk premium. As long as the security market line stays stable, its slope which is the market risk premium will also remain constant.

4. The equilibrium price of a stock would be such that given the beta of the stock its expected rate of return would fall on the security market line. Therefore, if a stock is priced such that given its beta its expected rate of return is below the security market line, it is overpriced and investors should sell it, and the reverse holds if a stock's risk-return profile falls above the security market line.

5. The market portfolio itself, which has a beta of one, is an efficient portfolio and falls on the security market line.

Markowitz model and CAPM, together with their assumptions and implications, will be described in details in the literature review chapter. As the empirical validity of the above findings of the CAPM is still controversial, in this study they will be viewed as hypotheses to be tested against empirical data. In Chapter 3 of this dissertation a research design for testing the standard CAPM hypotheses against recent data will be suggested and a multifactor CAPM will be proposed to be tested.

Definitions of Terms

Major terms that are frequently used in this study and their operational definitions are specified below. More technical terms that are related to some specific parts of the study will be defined as they are referred to.

Investment cash flows: The cash equivalent of benefits from an investment, measured in dollar terms.

Expected rate of return: Benefits required, in excess of the initial investment, from an investment expressed as the percentage of the initial investment and for a specific holding period. As for the common stock of a corporation, the expected rate of return of common stock at time t is measured by forecasting the price of the stock at time $(t+1)$ and the dividends to be received at time $(t+1)$ from owning the stock and dividing the sum of these two forecasts less the stock price at time t by the price of the stock at time t. Symbolically, this is written as:

$E(R_t) = \frac{E(P_{t+1} + D_{t+1}) - P_t}{P_t}$, where E stands for expectation, R stand for rate of return, P stands for price, D stands for dividend, and t stands for time.

Realized rate of return: The rate of return actually achieved on an investment over a specific holding period. For common stocks, the realized rate of return at time t is measured by: $R_t = \frac{(P_t + D_t) - P_{t-1}}{P_{t-1}}$.

Beta (β): In a single factor model, this is the sensitivity of the rate of return of an asset to the rate of return of the whole universe of the risky assets. However, for practical purposes usually a broad index like S&P 500 Stock Index is used as the proxy for the universe of risky assets. In a multifactor model there will be more than one beta, each of which measures the sensitivity of the rate or return of an asset to changes in some specific risk factor. For common stocks, the betas are measured by estimating the regression coefficients for time-series data relating the rates of return on the common stock to the relevant risk factors.

Risk or volatility of returns: The degree of variability of the rates of return of an asset and is measured by standard deviation of the rates of return of the asset through some time periods. The expected risk or volatility of a stock is measured by forecasting the distribution of future cash flows from the stock, computing the distribution of expected rates of return of the stock, and calculating the standard deviation of those expected returns. The actual risk or volatility of a stock is measured by calculating the standard deviation of its holding period rates of return for some historical time periods.

Implied volatility: Ameasure of expected or future volatility of a security's or of the whole stock market's rate of return derived from the Black-Scholes Option Pricing Model. Inclusion of this factor into modeling stocks' rates of return is this author's intended contribution to CAPM. This notion of implied volatility will be explained in more details in Chapter 3.

Risk-free rate: The rate of return with zero volatility which is earned on the risk-free asset. The proxy for the risk-free asset is US Government Treasury-Bills. The maturity chosen will depend on the holding period chosen in the study. Therefore, the measure of risk-free rate would be the rate of return on the relevant Treasury-Bill.

Risk premium: Risk premium of a stock, also called stock's excess return, is defined as the holding period rate of return of the stock less the risk –free rate of the same holding period.

Market risk premium: Also called market excess return, is the holding period rate of return on the overall stock market less the risk-free rate of the same holding period.

Financial leverage: A company's degree of indebtedness, measured by dividing total long-term debts (debts over one year maturity) of the company by its total assets.

Operating leverage: A company's level of fixed costs in relation to total costs of operation. As fixed costs are directly related to fixed assets, in this study the operating leverage is measured through dividing net fixed assets by total assets.

Capital market: The market in which corporate equity and long-term debt securities (those maturing in more than one year) are issued and traded.

Assumptions

As the primary purpose of this study is to test the CAPM hypotheses, initially the study will start with the same assumptions as those made by the CAPM. Because the rates of return and variation in the rates of return of the stocks are the result of investors' actions in the capital market, the assumptions of CAPM are stated in relation to (a) the investors' behavior and (b) the structure of the capital market. The assumptions of CAPM are:

1. Like the Markowitz model, CAPM assumes that only two characteristics of investment in securities are of relevance to investors; the expected rate of return and

the risk of securities. The expected rate of return is defined as the forecast of future pay-off from the investment in excess of initial investment divided by the initial dollar value of the investment and risk is defined as the probability of actual returns being different from expected return, measured by standard deviation of returns. The CAPM, as well as the Markowitz model, assume that investment risk is viewed with this perspective. In this sense investment decisions are made on the basis of only the first two moments of the probability distribution function of returns; the first moment, which is the expected or average rate of return, and the second moment, which is the variance of rate of return reflecting the amount of risk in the investment.

2 Investments are made by rational mean-variance portfolio optimizer investors who use the Markowitz model to select an efficient portfolio from the efficient frontier.

3. Investments in securities are made by investors who all have a similar economic view of the world and analyze securities in the same way. Therefore, estimates of probability distribution of securities' returns and of the expected rates of return, expected variance and covariance of returns, and expected future cash flows of all securities are identical for all investors. This assumption implies that all investors envision the same Markowitz efficient frontier portfolios and price securities according to the same method and on the basis of the same inputs. This assumption is usually referred to as *homogenous expectations* or beliefs assumption.

4. Like any other perfectly competitive market, the capital market consists of many buyers and sellers of securities, called the investors. The wealth of each individual investor is small as compared to the total wealth of all investors and therefore each

investor is a price-taker in the capital market. Although equilibrium prices are determined by the actions of all investors, the action of one individual investor by itself does not affect market prices.

5. Investment holding period is identical for all investors. This *single* holding period could be one month, one year, or any other time period. But whatever it is all investors are assumed to have homogenous holding period investment horizon.

6. Investments are limited to the universe of all publicly traded financial assets, like stocks, mutual funds, and bonds and to a risk free asset. Therefore, this assumption excludes investments in privately traded assets or investments in nontraded assets such as investments in education.

7. There is a risk-free asset in the capital market, that is, an asset with zero variance of returns, that all investors can lend or borrow any amount of that risk-free asset at an identical risk-free rate.

8. Investment in the capital market does not involve any transaction cost or does not result in any tax liability for the investors. This assumption ensures expected returns and variance of returns are the only factors that investors consider when selecting or rebalancing their portfolios.

The first assumption outlined above, that investors are risk averse and prefer less risk to more risk for the same expected return, is the core assumption of CAPM as well as of any investment theory proposed to date. Whether this assumption is realistic or not is a matter to be verified by behavioral psychologists and is not addressed in this study. Therefore, in this study the risk-averse assumption is taken for granted both for testing the standard (single-factor) CAPM and for testing the multifactor model. As will be discussed in the literature review, other scholars have shown that theoretical findings of the standard CAPM remains valid even if some of the model assumptions are dropped. Thus, in the test of hypotheses for standard CAPM all the assumptions made above are kept, as it does not theoretically make any difference to drop some of them. But, in the test of hypothesis for the multifactor model all the assumptions are dropped except the risk aversion assumption.

Scope and Delimitations

Theoretically the scope of CAPM, single-factor or multifactor, entails all risky assets for which there are markets. This will include stocks and bonds of public and nonpublic corporations, real estate, foreign exchange, gold and other precious metals, antiques, and so on. However, it is not practically possible to collect data and test the CAPM for the universe of all risky assets, nor such an attempt has ever been made. In this study, like most studies on CAPM, the target population is confined to all publicly traded companies in the US whose securities are traded in one of the organized stock exchanges or on NASDAQ National Market. Therefore, companies with stocks trading on the Pink Sheet, OTC Bulletin Board, or privately held corporations are not included in this study due to non-availability of adequate financial data on such companies. Characteristics, or variables, of the units of analysis that are studied are monthly closing stock prices, variability of monthly stock prices, dividends paid by the company to common stockholders, and some financial statement items including, asset size, financial leverage, and operating leverage. The study is focused on determining risk factors that affect the expected rates of return of common stocks in publicly traded corporations, finding the nature of the relationship between risk and return, and estimating the relevant coefficients in the derived relationships.

Limitations

As this research employs the causal-comparative design using existing data, the result of the study is affected by operational definitions of the concepts and by the way they are measured. The estimated risk premiums for the market, the market implied volatility, financial leverage, operating leverage, and other independent variables in the regression analysis depend on how these variables are defined and measured. Furthermore, in this study the S&P 500 Stock Index is chosen as the proxy for the market portfolio. Using other proxies for the market portfolio could affect the results reported in this study. As the empirical data used to test the hypotheses are related to the common stocks of public companies trading in the US stock market for the period January 1, 1995 to Dec 31, 2004, the conclusions of this study can not be generalized to assets other than stocks, to stocks of nonpublic companies, to the stocks trading in other countries stock markets, and to periods other than the sample period.

The time-series regression tests employed in this study are based on the assumptions of normality of monthly rate of returns and lack of serial autocorrelation in the residuals of the regression models. Departure from normality and existence of autocorrelation will cast doubt on the reliability of inferences from the estimated regression coefficients.

The Hypotheses

The hypotheses to be tested in this study pertain to the relationships between rates of return on stocks of public companies and relevant risk factors. Hypothesis one is to test the validity of standard CAPM for the sample period, January 1, 1995 to December 31, 2004. Hypotheses two, three, and four are this author's propositions to bridge the field of investment theory to the field of corporate finance theory. And hypothesis five is this author's proposed contribution to the field.

Hypothesis one: The market factor is the only risk factor that explains variation of returns across different stocks and the more sensitive a stock's rate of return with respect to the market factor, the higher the rate of return on the stock. This is the basic standard single-factor CAPM hypothesis, which states that the rates of return on stocks are linearly related to the rates of return of the market portfolio. Hypothesis one involves testing the following two regression models for significance of correlation and regression coefficients:

$$R_{jt} - R_{ft} = \alpha_j + \beta_j(R_{Mt} - R_{ft}) + e_{jt}$$

$$\overline{R_j - R_f} = \lambda_0 + \lambda_1 b_j + \lambda_2 \sigma^2(e_j) + e'_j$$

Using the language of test of hypothesis, the null and alternate hypotheses for the first equation of hypothesis one would be:

$$H_0 : \alpha_j, \beta_j = 0$$
$$H_1 : \alpha_j, \beta_j \neq 0$$

, and for the second equation of hypothesis one the null and the alternate would be:

$$H_0 : \lambda_0 = 0, \lambda_1 = \overline{R_M - R_f}, \lambda_2 = 0$$
$$H_1 : \lambda_0 \neq 0, \lambda_1 \neq \overline{R_M - R_f}, \lambda_2 \neq 0$$

where, R_{jt} is the realized rate of return on stock *j* during the month *t*, R_{ft} is the risk-free rate during the month *t*, and R_{Mt} is the rate of return on the market portfolio during the month t , $\overline{R_j - R_f}$ is average monthly risk premium of stock *j* during the study period (average monthly rate of return of stock *j* less average monthly risk-free rate during the study period of 120 months), e_{jt} is the error term for stock *j*'s rate of return in month *t*, and $\sigma^2(e_j)$ is the variance of stock *j***'**s error term during the study period.

Hypothesis two: Expected (average) monthly rate of return of small stocks is higher than expected (average) monthly rate of return of large stocks. This is based on the corporate finance proposition that investors regard small companies to be more risky than large companies as small companies face more *business risk* than the large companies. This proposition can be tested through the following null and alternate hypotheses:

$$H_0 : \mu(\overline{R}_j^{SAS}) \leq \mu(\overline{R}_k^{LAS})$$
$$H_1 : \mu(\overline{R}_j^{SAS}) > \mu(\overline{R}_k^{LAS})$$

where, $\overline{R}_j^{SAS}$ is average monthly rate of return of the small company *j* over the study period (average rate of return over 120 months), $\overline{R}_k^{LAS}$ is average monthly rate of return of the large company *k* over the study period, $\mu(\overline{R}_j^{SAS})$ is the mean of all small companies average rate of returns

Hypothesis three: Expected (average) monthly rate of return of high financial leverage stocks is higher than expected (average) monthly rate of return of low financial leverage stocks. This is based on the corporate finance proposition that investors regard high financial leverage companies to be more risky than low financial leverage companies as high financial leverage companies face more *financial risk* than the low financial leverage companies. This proposition can be tested through the following null and alternate hypotheses:

$$H_0 : \mu(\overline{R}_j^{HFL}) \leq \mu(\overline{R}_k^{LFL})$$
$$H_1 : \mu(\overline{R}_j^{HFL}) > \mu(\overline{R}_k^{LFL})$$

where, $\overline{R}_j^{HFL}$ is average monthly rate of return of the high financial leverage company ***j*** over the study period (average rate of return over 120 months), $\overline{R}_k^{LFL}$ is average monthly rate of return of low financial leverage company *k* over the study period, $\mu(\overline{R}_j^{HFL})$ is the mean of all high financial leverage companies average rate of returns, and $\mu(\overline{R}_j^{LFL})$ is the mean of all low financial leverage companies average rate of returns.

Hypothesis four: Monthly rate of return of high operating leverage stocks is higher than monthly rate of return of low operating leverage stocks. This is based on the corporate finance proposition that investors regard high operating leverage companies to be more risky than low operating leverage companies as high operating leverage companies face more *business risk* than the low operating leverage companies. Hypothesis four can be tested through the following null and alternate hypotheses:

$$H_0 : \mu(\overline{R}_j^{HOL}) \leq \mu(\overline{R}_k^{LOL})$$
$$H_1 : \mu(\overline{R}_j^{HOL}) > \mu(\overline{R}_k^{LOL})$$

where, $\overline{R}_j^{HOL}$ is average monthly rate of return of the high operating leverage company *j* over the study period (average rate of return over 120 months), $\overline{R}_k^{LOL}$ is average monthly rate of return of low operating leverage company *k* over the study period, $\mu(\overline{R}_j^{HOL})$ is the mean of all high operating leverage companies average rate of returns, and $\mu(\overline{R}_j^{LOL})$ is the mean of all low operating leverage companies average rate of returns.

Hypothesis five: This hypothesis is this author's proposed contribution to the field. It is a multifactor model and posits that (a) the expected rate of return of any stock is linearly dependent on five risk factors, the market return, the market implied volatility, the company's asset size, the company's financial leverage, and the company's operating leverage and (b) expected rate of returns across cross section of stocks are linearly related to coefficients of risk factors estimated in part (a).

Part (a) of hypothesis five is tested through solving the regression equation:

$$R_{jt} - R_{ft} = \alpha_j + \beta_j^M (R_{tM} - R_{ft}) + \beta_j^{IV}(IV_{t-1}) + \beta_j^S(SLL_t) + \beta_j^{FL}(HFLLF_t) + \beta_j^{OL}(HOLLO_t) + e_{jt}$$

And the null and alternate hypotheses:

$$H_0 : \alpha_j, \beta_j^M, \beta_j^{IV}, \beta_j^S, \beta_j^{FL} = 0$$
$$H_1 : \alpha_j, \beta_j^M, \beta_j^{IV}, \beta_j^S, \beta_j^{FL} \neq 0$$

where, the β_j's are the responsiveness of stock *j* rate of return to the specified risk factors.

Part (b) of hypothesis five is tested through solving the regression equation:

$$\overline{R_j - R_f} = \lambda_0 + \lambda_1 b_j^M + \lambda_2 b_j^{IV} + \lambda_3 b_j^S + \lambda_4 b_j^{FL} + \lambda_5 b_j^{OL} + e_j$$

And the null and alternate hypotheses:

$$H_0 : \lambda_0 = 0,\ \lambda_1 = \overline{R_M - R_f},\ \lambda_2 = \overline{IV},\ \lambda_3 = \overline{SLl},\ \lambda_4 = \overline{HFLLF},\ \lambda_5 = \overline{HOLLO}$$

$$H_1 : \lambda_0 \neq 0,\ \lambda_1 \neq \overline{R_M - R_f},\ \lambda_2 \neq \overline{IV},\ \lambda_3 \neq \overline{SLL},\ \lambda_4 \neq \overline{HFLLF},\ \lambda_5 \neq \overline{HOLLO}$$

where, the b_j's are estimates of the β_j's found from the first regression equation.

These hypotheses are restated in chapter 3 and tested in chapter 4.

The Significance of the Study

The findings of this research have important implications for social change. The outcome of this study can change the way individual and institutional investors make investment decisions, can change the process that mutual funds select their portfolio managers and investment advisors, can change the rate determination for public utility companies, and can change capital budgeting and capital expenditure decisions of public corporations. These changes in turn could lead to significant changes in the resource allocation in the economy, in the economy's production capacity and production composition, and in the employment structure of the society.

The outcome of this study can assist investors and portfolio managers to identify undervalued or overvalued stocks and portfolios and make appropriate investment decisions. It also enables institutions to evaluate performance of their portfolio managers. Performance measurement is especially important in the mutual funds performance disclosures; and the outcome of this study can provide the ground for mutual funds to assess performance in connection with the amount of risk taken.

Finally, certain findings of the study can be of particular interest to institutions that invest for the purpose of meeting specific future liabilities. The proposition that there is a stable average risk premium from investing in the general market for longer time periods can help insurance companies, foundations, defined benefit plans, and even the Social Security Administration to invest part of their funds in the market portfolio and better meet their future liabilities.

Summary

This research is about the existence and nature of the relationship between rates of return on stocks and the risk factors for which investors in the stocks want to be compensated. This will be done by positing and testing five hypotheses with regard to the relationship between rates of return of stocks and five risk factors, market return, market implied volatility, size, financial leverage, and operating leverage.

Chapter 2, the literature review chapter, starts with a review of the Markowitz portfolio selection theory, from which modern portfolio theory originates and forms the conceptual foundation of the CAPM. As the CAPM is at present viewed as the most popular investment theory both by the academicians and by the investment professionals, the literature review entails a comprehensive review of CAPM, analyzing it both from the theoretical viewpoint and in terms of its empirical verification by scholars of the field. The section on CAPM will include the assumptions of CAPM, its analytical fidings, extensions and modifications of the model, and empirical tests of CAPM conducted by scholars of the field. The Arbitrage Pricing Theory (APT), postulated by Ross (1976) as an alternative investment theory challenging CAPM will also be reviewed, compared, and contrasted with CAPM in Chapter 2.

The methodology and research design of the study is described in chapter 3. The chapter starts with an overview of the type of research design employed and the nature of the relationships to be studied. The section on sample in chapter 3 deals with describing the target population, type of data, sources of data, sampling frame, data collection, pilot study, and sampling design. The final section of chapter 3 will be about type of data analysis, which will include detailed description of the variables of the model and their operational definitions, the hypotheses to be tested, and the type of tests. Analysis of the data and test of the five stipulated hypotheses are covered in chapter 4. Results and findings of data analysis and test of hypotheses are also reported in chapter 4. Finally, conclusions derived from the study and recommendations for further research are specified in chapter 5.

CHAPTER 2:

LITERATURE REVIEW

Introduction

This literature review is centered around scholarly writings about the capital asset pricing model (CAPM), including the standard CAPM as originally advanced by Sharpe (1964), Linter (1965), and Mossin (1966); its modification and extensions by other scholars; and major empirical verifications of the model's validity. However, the CAPM is built upon and retains some of the assumptions as well as the findings of its predecessor, the Markowitz portfolio theory. To understand the CAPM, it is essential to know Markowitz portfolio theory and, therefore, a literature review on CAPM needs to be preceded by a discussion of Markowitz model.

This literature review starts with a detailed analysis of Markowitz portfolio theory and its relevance to the hypotheses and problems of this study. There will then be an analytical review of the standard CAPM, followed by a study of major modifications of the model brought about by other scholars. An examination of main empirical verifications for CAPM validity and their findings will be next task to be covered in this literature review. Finally, the arbitrage pricing theory (APT), which was advanced as the rival to CAPM, will be reviewed and analyzed.

Markowitz Portfolio Selection Theory

Modern portfolio theory (MPT) started with the pioneering work of Harry Markowitz in 1952 which earned him the 1990 Noble Prize in economics because of the enormous impact of his theory on investment management thereafter. On the basis of Markowitz (1952), he is often called the father of Modern portfolio theory, though he believes A. D. Roy can claim an equal share of this honor (Markowitz, 1999, p.5). As the name implies, Markowitz portfolio selection theory is not about investment in a single asset but rather it is a theory that explains how investors construct, or should construct, a diversified portfolio of different assets. Thus, the notion of *diversification* is at the core of Markowitz theory. According to Markowitz (1952) the process of selecting a portfolio consists of two stages. The first stage starts with observation of historical performances of and experiences with available securities and ends with formation of beliefs or expectations about their future performances. The second stage starts with the relevant beliefs or expectations about future performances and ends with selection of the portfolio. Markowitz (1952) model is concerned with the second stage, that is, it assumes that expectations about future performances are already formed, with reference to past performances or otherwise. And then those expectations are used as known inputs of the model to solve for the unknown parameters (pp. 77-78).

Markowitz (1952) starts by rejecting the economist's rule of discounted expected return maximization, both as a hypothesis explaining investors' behavior and as a maxim to guide their behaviors, on the ground that even if risks of individual securities are included in the discounting factor the rule always leads to choosing one single security; the one with the highest discounted expected returns. Such a rule implies that the investor should always put all his or her money in one security only, the one with highest discounted expected returns. Markowitz aptly argues that the discounted expected return maximization rule disregards the phenomenon of diversification which is both observed in real life and is sensible. The rule can never imply that there exists a diversified portfolio that is preferred to all non-diversified portfolios, including to the security with maximum expected returns. Therefore, because expected return maximization rule disregards the superiority of diversification, Markowitz rejects it both as a hypothesis and as a maxim. Instead he proposes the rule that investors *do* or *should* consider expected returns a desirable thing and variance of returns an undesirable thing. Markowitz (1952) does not use the tem investment *risk* in his original work. However, the variance of returns used in Markowitz model, or its square root the standard deviation of returns, is now regarded as the measure of investment risk by most theoreticians and practitioners in the field of investment management. But even with diversification, the portfolio with maximum expected return is not necessarily the one with minimum variance. There is a pace at which the investor can gain expected return by taking more variance or reduce variance by giving up some expected return. Markowitz calls this trade-off between expected return and variance the *expected return-variance*, or *E-V*, rule and argues that this rule provides a better explanation of and guide to investment decision-making than the expected return maximization rule while it distinguishes *investments* from *speculative* behavior. Markowitz furthermore demonstrates that the *E-V* hypothesis not only implies diversification, but it also implies the *right* diversification for the right reasons. Investing in many securities by itself is not the right diversification, because securities' returns might have high correlations with one another. A portfolio with 100 technology securities from the technology industry is not as diversified as, and will carry more variance, than a portfolio with the same number of securities from different industries (pp. 78-90). As will be described in the following paragraphs, Markowitz model provides the guideline and the methodology for an investor to create an optimal portfolio on the basis of his or her risk tolerance, the expected returns of securities, the variances (or standard deviations) of securities' returns, and the covariances or correlations between security returns.

In Markowitz's portfolio selection model the universe of risky investments consists of *n* risky securities from which investors can select and construct their portfolios. The one-period holding rate of return from investing in security *i* is R_i, where each R_i is a random variable with known probability distribution. Knowing the probability distribution functions of each R_i enables one to calculate expected value of returns and expected value of variances of returns for every security as well as the expected values of covariances of returns or correlations between any two securities. Assuming that future probabilities of returns are the same as the past ones, the expected values of returns and their expected variances and covariances can be estimated from historical data of one-period rates of return. Therefore, in Markowitz model the following information is available to all investors at any point in time about the available securities in the market:

$\mu_i = E(R_i)$ = Expected rate of return of security *i* = average of historical returns of security *i*,

σ_i^2 = Expected variance of returns of security ***i*** = variance of historical returns of security *i*

ρ_{ij} = Expected correlation coefficient between returns on security *i* and returns on security *j* = historical correlation coefficient between returns on security *i* and returns on security *j*.

Now with this information, an investor with a specific amount of wealth, or funds, to invest in securities must decide what proportion of his or her wealth or funds to invest in each security. Denoting the percentage of fund to be invested in security ***i*** by$\boldsymbol{w}_i$, the portfolio selection problem leads to some sort of solution of the following set of equations:

Expected return of the portfolio=

$$\mu_p = w_1\mu_1 + w_2\mu_2 .... + w_n\mu_n = \sum_{i=1}^{n} w_i\mu_i \tag{1}$$

Expected variance of return of the portfolio=

$$\sigma_p^2 = (w_1^2\sigma_1^2 + ... + w_n^2\sigma_n^2) + 2(w_1w_2\rho_{12}\sigma_1\sigma_2 + ...w_iw_j\rho_{ij}\sigma_i\sigma_j ... + w_1w_n\rho_{1n}\sigma_1\sigma_n)$$

Or:

$$\sigma_p^2 = \sum_{i=1}^{n} w_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} w_iw_j\rho_{ij}\sigma_i\sigma_j \tag{2}$$

And

$$w_1 + w_2 + ...w_n = 1 \tag{3}$$

where, the weights in Equation 3 are all non-negative and less than or equal to one when short-sale is not permitted. When short-sale is allowed some weights can be negative and some can be greater than one. The securities shorted have negative weights and the securities purchased with the proceeds of short-sale have weights greater than one, but the sum total of all weights still adds up to one.

When there are many assets in the portfolio it is usually preferred to write Equations 1-3 in matrix form which makes it easier to calculate portfolio's expected return and variance by employing techniques of matrix algebra. Following Benninga (2000, pp. 133-140) and making the following notations:

$$\boldsymbol{W} = \begin{bmatrix} w_1 \\ w_2 \\ . \\ w_n \end{bmatrix}, \qquad \boldsymbol{R} = \begin{bmatrix} \mu_1 \\ \mu_2 \\ . \\ \mu_n \end{bmatrix}, \ \boldsymbol{\rho} = \begin{bmatrix} \rho_{11} & \rho_{12} & . & \rho_{1n} \\ \rho_{21} & \rho_{22} & . & \rho_{2n} \\ . & . & . & . \\ \rho_{n1} & \rho_{n2} & . & \rho_{nn} \end{bmatrix}, \text{ and } \boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & \\ . & . & . & . \\ 0 & 0 & 0 & \sigma_n \end{bmatrix}$$

Then Equations 1-3 can be expressed in matrix form as:

$$\mu_p = \boldsymbol{W}^{\boldsymbol{T}} \boldsymbol{R} \tag{5}$$

$$\sigma_p^2 = \boldsymbol{W}^{\boldsymbol{T}} \sigma\rho\sigma \boldsymbol{W} \tag{6}$$

$$\boldsymbol{W}^{\boldsymbol{T}} \boldsymbol{I} = 1 \tag{7}$$

where, $\boldsymbol{W}^{\boldsymbol{T}}$ is the *transpose* of **W** and ***I*** is the *unit* column vector with *n* rows.

The set of Equations 1-3 can be converted into one equation expressing a relationship between the portfolio variance $\sigma_p^{\,2}$, the portfolio mean μ_p, and some of the weights. To do so, first one of the weights *w* can be expressed in terms of other *w*'s and substituted into Equations 1 and 2. Then from Equation 1 another of the *w*'s can be expressed in terms of μ_p and then substituted into Equation 2. The result would be a quadratic relationship between portfolio's risk and portfolios' expected rate of return, expressed in the following general form:

$$\sigma_p^2 = a + b\mu_p + c\mu_p^2 \tag{8}$$

where, ***a*****,** *b*, and *c* contain *(n-2)* of the *w***'**s. With σ_p on the x-axis and μ_p on the y-axis, plot of Equation 8 would be a series of hyperbolas, each of which represents a specific combination of the values of the portfolio weights, *w***'**s. The set of all such possible portfolios was called the *feasible set* by Markowitz. Simply putting it, the feasible set consists of all portfolios for which the sum of the *w*'s adds up to 100%. Markowitz (1952) demonstrated that if for any level of expected return μ_p one finds the portfolio with lowest risk σ_p on one of the hyperbolas derived from Equation 8, and connects all the resulting points then another hyperbola will emerge that encompasses or envelopes all the other hyperbolas. This resulting set of envelope portfolios, the overlapping hyperbola, represents all the portfolios that have the lowest risk for the same level of expected returns. The upper part of the envelope represents portfolios that not only have the lowest risk for the same expected return but they also have the highest expected return (μ_p) for the same level of risk (σ_p). These portfolios lying on the upper part of the encompassing hyperbola was called the *mean-variance efficient portfolios* by Markowitz. In the literature on Modern Portfolio Theory, the mean-variance efficient portfolios, that is, the upper part of the envelope portfolios is referred to as the *efficient frontier* and the whole envelope is called *minimum variance frontier*. The lower part of the envelope does not contain efficient portfolios because for any level of risk it is always possible to move to the upper part and gain a higher expected return. The vertex of the envelope, that is, where the convexity of the envelope changes, has the minimum of all minimum variances and therefore is called the *global minimum variance* portfolio. It can also be shown that when short-sale is not permitted individual assets lie on the envelope, both on the efficient frontier and on the none-efficient part of the envelope. But when short sale is permitted all individual assets will be inside the encompassing hyperbola and thus no individual asset can be mean-variance efficient when there is the possibility of short sale (Bodie, Kane, & Marcus, 1993, pp. 214-216). These ideas are demonstrated graphically in Figure 1 below:

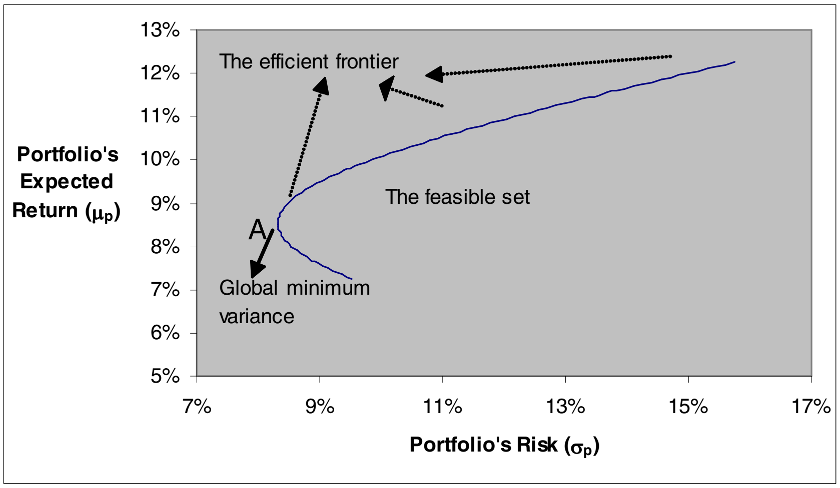

Figure 1. The efficient frontier and optimal portfolio selection.

The area inside the hyperbola contains all possible portfolios; it is the feasible set. All the points outside of the hyperbola are not feasible portfolios. Every point in the feasible set represents specific values of the *w*'s that satisfy Equation 3. Any point on the hyperbola, or envelope, represents a portfolio that has minimum variance for a specific value of expected return. The points on the upper part of the hyperbola starting from the vertex *A* represent the efficient portfolios and the whole upper part of the hyperbola is the efficient frontier. Point *A* represents an efficient portfolio with *minimum* risk and is the global minimum variance portfolio. It is interesting to note that as long as the securities are not all having perfectly positive correlation with one another, this minimum risk portfolio has less risk than the least risky security while its expected return which is the weighted average of all securities' returns is higher than the return of the lowest return security. As can be seen from Figure 1, for any specific level of risk that an investor is ready to take the corresponding portfolio on the efficient frontier yields the highest expected return, and for any specific expected return that an investor desires, the portfolio on the efficient frontier contains the least risk for the investor. Therefore, according to

Markowitz (1952. pp. 80-89) rational investors *do* and *should* select a portfolio from the efficient frontier. All that the investor needs to do is to decide how much risk he or she is willing to take and then the efficient frontier provides him or her with the optimal portfolio for that level of risk.

The concept of efficient frontier is by far the most significant contribution to investment theory and is widely employed by mutual fund companies to construct efficient portfolios. However, given the set of enormous feasible portfolios that can be constructed out of available assets, Markowitz model needed some refinements to be useful in practice. Black's Theorem provides this facility. Black (1972, pp. 46-49) proved, mathematically, that if two portfolios *A* and *B*, represented by vector weights $\boldsymbol{W_a}$ and $\boldsymbol{W_b}$ are on the minimum variance frontier (the envelope), then any portfolio with the weights: $\lambda \boldsymbol{W_a}$ and $(1-\lambda)$ $\boldsymbol{W_b}$**,** where λ is any arbitrary number, will also be on the minimum variance frontier. Therefore, if one finds two minimum variance or efficient portfolios, then the whole envelope of minimum variance portfolios can be constructed by giving various numbers to λ and graphing the expected returns of the resulting portfolios against their standard deviations. To find one efficient portfolio, say portfolio *A***,** one can assume any specific portfolio standard deviation σ_p, say 4%, and then set up an optimization problem to *Maximize* μ_A from Equation 1 subject to σ_p^2 from Equation 2 becoming 4%, and the portfolio weights summing up to one. This optimization problem can be solved using basic differentiation from calculus algebra or computer programs like the *solver* feature in Microsoft Excel. Repeating the same procedure for another assumed value of σ_p, a second efficient portfolio can be found and then employing Black's theorem the whole envelope can be generated.

In addition to the notion of mean-variance efficient portfolios, Markowitz's theorem has two other important theoretical implications for the practice of portfolio management. One implication, easily derived from the formula for portfolio variance (Equation 2) is that the lower the correlations between stocks in a portfolio the lower would be total risk of the portfolio. This is evident from the second term in the right-hand side of Equation 2, which shows that even if the variances of individual securities are kept the same, lower correlations lead to lower total portfolio's variance. This conclusion is very important for the practice of portfolio management. It implies that from the portfolio's perspective, the risk of an individual asset should not be assessed by its own variance but rather by how much it adds to the portfolio's total risk when it is added to a portfolio. Therefore, adding a high risk-high return security to a portfolio will increase portfolios' expected return, according to Equation 1, while if it has low correlations with other securities in the portfolio it will relatively add less risk to the portfolio or even could reduce total risk if it has negative correlations with some of the securities in the portfolio. The second implication also derived from Equation 2 but not as evident as the first one, is about the benefit of *diversification*; the more the number of securities in the portfolio the lower would be the total risk of the portfolio. This conclusion is usually proved mathematically by considering the special case where all the weights in the portfolio are equal, that is, when all the w_i's are equal to *1/n*. In this case the portfolios' variance $\sigma_p{}^2$ from Equation 2 can be reduced to:

$\sigma_p^2 = \sum_{i=1}^{n} \frac{1}{n^2}\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j=i+1}^{n} \frac{1}{n^2}\rho_{ij}\sigma_i\sigma_j$, which, given that there are n variances and $n(n-1)/2$ correlations, this can be simplified to:

$$\sigma_p^2 = \frac{1}{n}\overline{\sigma^2} + \frac{n-1}{n}\overline{Cov} \tag{9}$$

where, $\overline{\sigma^2}$ is the arithmetic average of variances of returns of individual securities and $\overline{Cov}$ is the arithmetic average covariances of returns between every two securities in the portfolio. The first term in Equation 9 above represents average risk *specific* to individual securities and as the number of securities in the portfolio, *n,* increases this specific risk decreases and ultimately becomes zero. Thus, adequate diversification eliminates specific risk of securities, or as usually described in the literature, specific risk can be *diversified away*. The second term in Equation 9*,* however, cannot be diversified way. When *n* becomes very large the coefficient of the second term approaches *1* and therefore total portfolio risk approaches the average covariance of the securities in the portfolio. Securities' returns covary when there are some general factors that affect the returns of all securities and that is why the second term in *Equation 9* is referred to as the *systematic* risk of the portfolio. Systematic risk is *not diversifiable* and its size depends on the degree of correlations between securities' returns. If on average the correlations between securities' returns turn out to be zero, then the second term in Equation 9, that is the systematic risk, will also be zero with adequate diversification. In this case one might create a fully hedged portfolio through diversification. On the other hand, if on average the securities in the portfolio show perfect positive correlations then average covariance becomes the same as average variance, that is $\overline{Cov} = \overline{\sigma^2}$, and the systematic risk equates average variance of the securities' return. In this case diversification has no risk reduction benefit no matter how many securities are added to the portfolio (Bodie, Kane, & Marcus, 1993, pp. 232-237).

Optimal Portfolio Selection and the Utility Indifference Curves

Markowitz's efficient frontier provides the set of portfolios with highest expected returns for any level of desired risk. According to Markowitz model, an individual investor selects one portfolio from the efficient set by deciding the amount of risk, measured by the standard deviation of returns, which he or she wants to take. But how does an investor decide on the amount of risk to take? Economists answer this question by introducing the notion of *utility* into the investment decision making. The propositions that investors desire to have more expected return and less expected variance of return are translated into a mathematical utility function. To obtain the same welfare or utility from different investments, *risk-averse* investors *trade-off* more risk for more expected return, that is, the higher the risk of an investment the higher the rate of return investors will require from that investment. These ideas are expressed mathematically by the following utility function:

$$U_p = E(R_p) - a.Var(R_p) = \mu_p - a\sigma_p^2 \qquad (10)$$

where U_p is the utility obtained from investing in a portfolio that has expected return of μ_p and expected variance of σ_p^2 and a is a positive number which denotes the degree of risk aversion of the investor. More risk-averse investors are represented by high values of a than less risk-averse investors. Equation 10 indicates that if expected return increases utility will increase, if variance of return increases utility will decrease, and vise-versa. Equation 10 also shows that to keep the portfolio's utility at a constant value, there must be a trade-off between expected return and variance. This can be demonstrated graphically by rewriting Equation 10 expressing μ_p as a function of σ_p and a constant utility value U**.** and graphing μ_p against σ_p. The result would be a convex curve called *utility indifference curve* which represent various combinations of μ_p and σ_p that yield the same amount of utility. For every constant value assigned to U_p a separate utility indifference curve will result. Therefore, every investor will have a set of distinct parallel utility indifference curves, each of which represents all possible combinations of μ_p and σ_p that yield a specific amount of utility and each indifference curve yields a higher utility than the indifference curves below it. Because the investor has preference for more utility, he or she will want to be on the highest possible utility indifference curve. But, according to Markowitz's model the investor also wants to be on the efficient frontier. To achieve both objectives of being on the efficient frontier and on the highest utility indifference curve leads the investor to select the portfolio represented by the point of *tangency* of the efficient frontier with his or her highest possible utility indifference curve. This is the *optimal* portfolio for the investor. Because different investors have different degrees of risk aversion and thus different utility indifference curves, the optimal portfolio is different for different investors (Bodie, Kane, & Marcus, 1993, pp. 142-216). These ideas are demonstrated in Figure2 below:

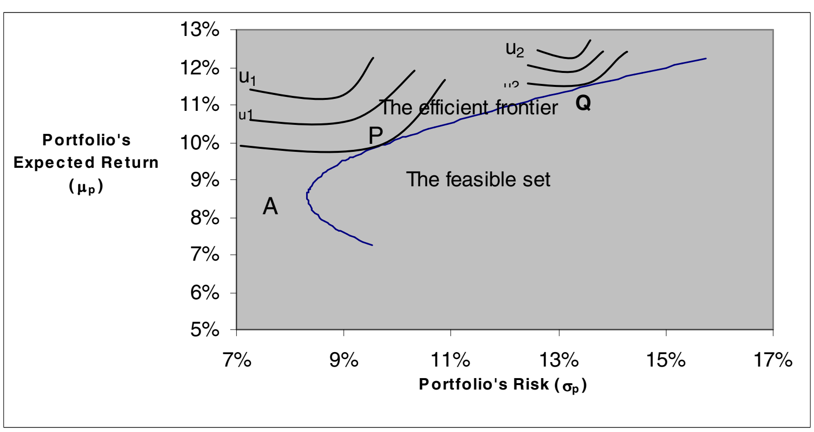

Figure 2. Efficient frontier, utility indifference curves, and optimal portfolio selection.

As shown in Figure 2, a more risk-averse investor whose utility indifference curves are represented by the set of U_1 curves will select the portfolio represented by point *P* on the efficient frontier while an aggressive investor with utility indifference curves U_2 will choose portfolio *Q* from the efficient frontier.

Optimal Portfolio Selection and the Risk Free Asset

James Tobin contributed to Markowitz portfolio selection theory by introducing the role of cash or a risk-free financial asset in the process of optimal portfolio selection and eventually in determination of equilibrium prices in the capital market. Tobin (1958, pp. 65-84) considered the case where the investor allocates his or her wealth between a risk-free asset and one of the portfolios on the efficient frontier. Subsequently Sharpe (1964, pp. 425-442) generalized Tobin's idea of the risk free asset by assuming that all investors can both lend (invest) or borrow at the same risk-free rate. This idea of investing in a portfolio on the efficient frontier together with the possibility of lending and borrowing at the risk-free rate led to a striking contribution to Markowitz's portfolio theory and formed the basis for the development of capital asset pricing model (CAPM). The argument is that any mixture of risk-free asset with a specific portfolio on the efficient frontier will lie along the line that joins the risk-free rate, on the y-axis, to the portfolio on the efficient frontier. This idea is demonstrated in Figure 3 below.

In Figure 3, point *F* on the vertical axis represents the rate R_f at which the risk-free asset can be lent or borrowed and the point *Q* is a portfolio on the efficient frontier. The risk-free asset has zero variance and zero correlation with portfolio *Q*. Therefore, in an asset allocation between portfolio *Q* and the risk free asset with weights *w* and *(1-w)* respectively, Equations 1 and 2 will be reduced to: $\mu_p = (1\text{-}w)\ R_f\ + w\ \sigma_q$ and $\sigma_p = w\ \sigma_q$.

Eliminating the term *w* between these two equations gives the relationship between the expected return and risk of the portfolio consisting of various combinations of the risk-free asset and portfolio *Q* to be:

$$\mu_p = R_f + (\frac{\mu_q - R_f}{\sigma_q})\sigma_p \tag{11}$$

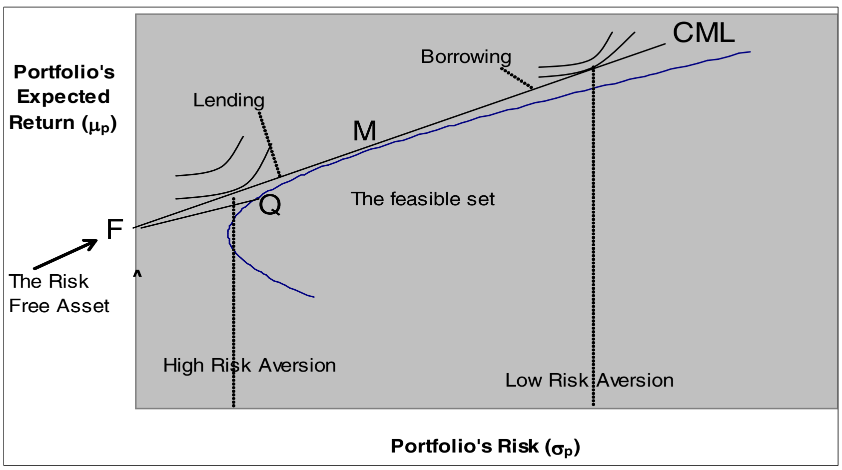

Figure 3. The capital market line (CML) and the global optimal portfolio

Equation 11 represents a line with the y-intercept R_f and slope $\frac{\mu_q - R_f}{\sigma_q}$. Looking at Figure 3 reveals that Equation 11 is exactly the equation of the line that joins point *F* to point *Q*. Therefore, all combinations of the risk free asset and portfolio *Q* are along the line *FQ*. The slope of this line as portrayed above is portfolio *Q's* return in excess of the risk free rate per unit of portfolio *Q's* risk. Now, rearranging the terms in Equation 11 one can get:

$$\frac{\mu_p - R_f}{\sigma_p} = \frac{\mu_q - R_f}{\sigma_q} \tag{12}$$

In other words, all the portfolios along the line *FQ*, although represent different returns and different risks, but they have the same amount of portfolio's excess return per unit of portfolio's risk, which are all equal to the excess return per unit of risk of portfolio *Q*. All the lines attaching point *F* to a point on the efficient frontier are called *capital allocation lines* (CAL), because each one of them represents allocation of funds between the risk-free asset and a specific portfolio on the efficient frontier for investors with different degrees of risk aversion who want to obtain a specific level of excess return per unit of risk.

In developing the CAPM, Sharpe (1964, pp. 433-436) proposed that rational investors will try to maximize excess return per unit of risk they take and, therefore, will try to be on a line similar to *FQ* but with the highest possible slope. This is achieved by finding the portfolio *M* on the efficient frontier such that the line *FM* becomes tangent to the efficient frontier. All portfolios consisting of a mix of portfolio *M* and the risk free-asset are positioned along the line *FM* and have the same highest possible excess return per unit of risk. Therefore, portfolio *M* is the optimal risky portfolio for all investors irrespective of their risk aversions and all investors want to be somewhere along the line *FM*. Investors that are more risk-averse and conservative will invest part of their wealth in portfolio *M* and lend (invest) the rest at the risk free-rate. Aggressive investors that are less risk-averse would invest more than their wealth in portfolio *M* by borrowing at the risk-free rate and investing the proceeds in portfolio *M*. All the points along the line *FM* and to the left of point *FM* represent portfolios that investors are lending part of their wealth at the risk-free rate and all the points to the right of point *M* along the line *FM* are portfolios of aggressive investors that borrow at the risk-free rate and invest more than their wealth in portfolio ***M*.** An investor decides where to be along the line *FM* by finding the point of tangency of his or her utility indifference curve with the line *FM*. This theoretically important line that passes through the risk-free asset and is tangent to the efficient frontier is called the *capital market line* (CML). The implication of this argument is very striking. There is only *one* optimal portfolio that all investors, regardless of their degrees of risk aversion, are interested in; and that is the optimal portfolio *M*. The degree of risk aversion of an individual investor comes into play only in the selection of a desired point on CML. Therefore, portfolio investment consists of two separate tasks; the *investment* task and the *financing* task. The investment task is to find the universal optimum portfolio *M* and the financing task is to decide to either borrow or lend to attain a preferred point on the CML. In investment theory, this separation of investment decision from financing decision is called the *separation theorem*.

The first task of portfolio investment, that is, the task of finding the global optimal portfolio ***M*** and thus the CML, can be solved by finding the weights *w*'s that maximize $S_M = \frac{\mu_M - R_f}{\sigma_M}$ subject to Equations 1 to 3 where, S_M is the slope of CML. Once the global optimal portfolio and, therefore, the slope of CML is established, the investor should find the point of tangency of his or her highest possible utility indifference curve with CML; and that point determines whether the investor should lend or borrow at the risk-free rate to invest less or more in the global optimal portfolio *M*. Theoretically this can be done by equating the slope of the utility indifference curve with the slope of CML which solves for the point of tangency of the two. The slope of the utility indifference curve can be found by differentiating μ_p with respect to σ_p in Equation 10, treating *U* as a constant, which yields $2a\ \sigma_p$. Putting this equal to S_M ,the slope of CML, and considering that $\sigma_p = w\ \sigma_M$, where w and (1-w) are allocation weights between portfolio M and the risk free asset respectively, the weight w to be invested in portfolio M comes out to be

$$w = \frac{\mu_M - R_f}{2a\sigma_m^2} \tag{13}$$

It can be seen from Equation 13 that conservative investors with a high a value will invest a smaller portion of their wealth in the risky portfolio M and specifically w can be smaller or greater than 100% depending on the degree of a specific investor's risk aversion a.

Summary

Markowitz model, with its refinements, is essentially an optimization model and as such is more of a normative theory. Based on the assumption that investors are risk averse and prefer less risk to more risk for the same expected return, the model provides guidelines for the investors as to how to construct optimal portfolios given their knowledge of expected returns and risks of every security in the market and the benefits of diversification. However, the model does not explain how investors form their return expectations about individual securities in the first place. Therefore, Markowitz model can not serve as a theory of equilibrium prices in the capital market. This task is undertaken by the CAPM which will be reviewed and analyzed next.

Capital Asset Pricing Model

The CAPM is generally attributed to William Sharpe's 1964 article, for which he earned the Noble Prize, But Linter (1965) and Mossin (1966) independently derived similar theories. The CAPM is essentially a model that tries to explain how equilibrium prices of individual assets in the market are established. The model posits that for any individual asset, or portfolio of assets, there is a linear relationship between the expected rate of return of the asset and the risk of that asset. On the basis of this return- risk relationship investors first form their expectation of the rate of return they require from an asset and then discount the future expected pay-offs from that asset at that expected rate of return. If the market price of an asset is more (less) than that discounted amount then they sell (buy) the asset in the market until its price equates the discounted value; and the capital asset market will be at equilibrium when all assets trade at prices that reflect the discounted value of their expected future pay-offs. The problem of explaining equilibrium asset prices in the capital market, therefore, leads to the problem of explaining how investors form their expectations or requirements with respect to the rates of return of the assets; and this is what CAPM is all about. Sharpe (1964) points out that although prior theories try to explain investors' behavior in optimal portfolio selection, but none have attempted to extend their models to provide *market* equilibrium of asset prices under conditions of risk; and specifically they do not provide an explanation of how the price of an individual asset is related to the various components of its risk (pp. 425-427).

CAPM starts with Markowitz's portfolio selection model and by making some specific assumptions regarding both the capital market structure and the investors' behavior elaborates Markowitz's model into a model of equilibrium asset pricing by deriving a relationship between the expected rate of return of an individual asset with some measure of the risk of that asset. Therefore, in this study, analysis of CAPM will be organized as the following. First, there will be a full description of the assumptions of standard version of CAPM, as specified by Sharpe (1964), Linter (1965), and Linter (1966). Second, the reasoning that connects CAPM assumptions to its conclusions will be reviewed and analyzed. Third, some of the major amended versions of CAPM will be reviewed and discussed. Finally, major empirical tests of the CAPM will be reviewed and the implications of their findings and results will be used as partial support for developing the multi-factor CAPM proposed in this study.

The Assumptions of CAPM

Basically, CAPM is derived from and is a modified extension of Markowitz portfolio selection model with specific implications for equilibrium asset prices in the capital market. Therefore, like Markowitz model it assumes that participants in the capital market are rational risk-averse investors in the sense that they are mean-variance efficient portfolio optimizers. The assumptions of CAPM as specified by Sharpe (1964), Linter (1965), and Linter (1966).can be summarized as the following:

1. Like the Markowitz model, CAPM assumes that investors are interested in only two characteristics of securities when deciding to invest in them; the expected rate of return and the risk of securities. The expected rate of return is defined as the forecast of future pay-off or cash flows from the investment, net of the initial investment, divided by the initial dollar value of the investment. Risk is defined as the probability of actual returns being different from expected return and is measured by standard deviation of returns. The CAPM, as well as the Markowitz model, assume that investors view risk with this perspective. In this sense investors are concerned only with the first two moments of the probability distribution function of returns; the first moment, which is the

expected or average rate of return and the second moment, which is the variance of returns reflecting the amount of risk in the investment.

2 All investors are rational mean-variance portfolio optimizers and use Markowitz model to select an efficient portfolio from the efficient frontier.

3. All investors have similar economic view of the world and analyze securities in the same way. Therefore, all investors have identical estimates of probability distribution of securities' returns and of the expected rate of returns, expected variance and covariance of returns, and expected future cash flows of all securities. Furthermore, the rate of return of every security is normally distributed and therefore investors are only interested in the first two moments of securities' probability distributions. This assumption implies that all investors envision the same Markowitz efficient frontier portfolios and price securities according to the same method and on the basis of the same inputs. This assumption is usually referred to as *homogenous expectations* or beliefs assumption.

4. Like any other perfectly competitive market, the capital market consists of many buyers and sellers of securities, called the investors. The wealth of each individual investor is small as compared to the total wealth of all investors and therefore each investor is a price-taker in the capital market. Although equilibrium prices are determined by the actions of all investors, the action of one individual investor by itself does not affect market prices.

5. All investors plan for one identical holding period. This *single* holding period could be one month, one year, or any other time period. But whatever it is, all investors are assumed to have homogenous holding period investment horizon.

6. Investments are limited to the universe of all publicly traded financial assets, like stocks, mutual funds, and bonds and to a risk free asset. Therefore, this assumption excludes investments in privately traded assets or investments in nontraded assets such as investments in education.

7. There is a risk-free asset, that is, an asset with zero variance of returns, that all investors can lend or borrow any amount of the risk-free asset at an identical risk-free rate.

8. Investment in the capital market does not involve any transaction cost or does not result in any tax liability for the investors. This assumption ensures that expected returns and variance of returns are the only factors that investors consider when selecting or rebalancing their portfolios.

As they stand, these assumptions are an oversimplification of reality. But this does not necessarily mean that the conclusions and implications that are logically deduced from these assumptions are not valid. In fact, as will be discussed later on in this paper, other scholars have subsequently developed some modified versions of CAPM by dropping some of the assumptions that were made for the standard version of CAPM. The crucial thing about any theory or model, including the CAPM, is not to expect perfect validity of all the assumptions but rather it is to evaluate how well the model explains the reality and how well the predictions of the model are consistent with what actually takes place in the real world. To establish this, one needs to regard the model as a set of hypotheses and test it against actual data. Major tests of the CAPM will be discussed in this paper, but before that it is essential to know what conclusions the CAPM derives from these assumptions and what sort of analysis is made to reach those conclusions.

The Analytical Findings of CAPM

The Capital Market Line and the Market Portfolio

In developing the CAPM, Sharpe (1964) starts with the implications derived from the assumption that investors can lend and borrow unlimited amounts at the risk free-rate. As was discussed in the section *optimal portfolio selection with risk-free asset* in this study, this assumption leads to the conclusion that there is only one portfolio that *all* investors, no matter what their risk tolerances are, will regard as their optimal portfolio and will invest in that portfolio only. Investors' portfolios differ from one another only by the extent that they are mixed with different levels of lending or borrowing of the risk-free asset. This universally optimal portfolio is derived from the point of tangency of the efficient frontier with the capital allocation line (CAL) that passes through the risk-free rate on the vertical axis, as was shown in *Figure 3* above. The tangent line is called the capital market line (CML). The portfolio represented by the point of tangency is called, the *market portfolio* and is usually denoted by *M*.

The line CML in Figure 3 represents all portfolios containing various mixtures of the market portfolio *M* with lending or borrowing of the risk-free asset. An individual investor determines a suitable mix of portfolio *M* and the risk-free asset by finding the point of tangency of his or her highest utility indifference curve with the CML, or by just first deciding how much risk he or she wants to take and then spotting the appropriate point on the CML. According to Sharpe (1964), every investor wants to be on the CML because the reward-to-risk ratio on this line is the highest as compared to any other portfolio. The reward-to-risk ratio or the excess return per unit of risk of a security or of a portfolio is defined as the expected return of the security or portfolio minus the risk-free rate divided by the standard deviation of the security or the portfolio. Because all investors want to be on the CML, it follows that out of all portfolios on the efficient frontier investors only invest in the market portfolio *M*. combined with the risk-free asset. An investor with average degree of risk aversion will invest in portfolio *M* only, a high risk-averse investor will invest part of his or her wealth in portfolio *M* and will lend the rest at the risk-free rate, and a low risk-averse investor will invest more than his or her wealth in portfolio *M* through borrowing money at the risk-free rate. Sharpe (1964) further argues that because any security that trades in the capital market represents someone's investment and because every investor invests in portfolio *M*, it follows that portfolio *M* consists of the universe of all risky securities. And this is the reason why the portfolio *M* is called the market portfolio. Furthermore, equilibrium in the market implies that the weights of securities in the market portfolio *M* must be proportional to their market values, that is, the weight of a security in the market portfolio must be equal to its market price divided by the sum total of the market prices of all risky securities. The reason for this is that if an asset's weight in portfolio *M* is, for example, greater than what its market price justifies, excess demand for the asset will increase its price until its relative market value becomes consistent with its weight. Finally, because portfolio *M* contains all the risky assets it is a *completely diversified* portfolio. As was demonstrated in Markowitz portfolio selection theory, in a highly diversified portfolio the variance or the risk of the portfolio consists of only the return covariances of individual securities and the variances, or specific risks, of individual securities have negligible contribution to the portfolio's risk. Therefore, the market portfolio does not contain the *specific* or *unsystematic* risks of individual securities, and its risk σ_M is totally *systematic* caused by general macroeconomic factors reflected in the covariances between the returns of individual securities. As such, the variance of returns of the market portfolio M is due to macroeconomic factors that affect the returns of all securities and is not affected by firm specific factors. This is the core reasoning behind the legendary expected return-beta relationship of CAPM and will be discussed in the next section.

Given the proposition that at point *M* the CML is not only tangent to the efficient frontier, but is also tangent to the highest utility indifference curve of the average investor, one can work out the relationship between the expected return on the market portfolio and the average degree of investors' risk aversion. This is done by noting that at point *M* the slope of the utility indifference curve of the average investor is equal to the slope of the CML. The slope of the CML is given by:

$$S_M = \frac{E(R_M) - R_f}{\sigma_M}. \tag{14}$$

And the slope of the utility indifference curve at point *M* is obtained by differentiating the utility indifference function at point *M*, that is, by differentiating the function $E(R_M) = U_M + \bar{a} \times \sigma_M^2$ with respect to σ_M, which yield:

$$S_M = 2\bar{a} \times \sigma_M \tag{15}$$

From Equations 14 and 15, one gets:

$$E(R_M) - R_f = 2\bar{a}.\sigma_M^2 \tag{16}$$

Or alternatively:

$$\frac{E(R_M) - R_f}{2\sigma_M^{\ 2}} = \overline{a} \tag{17}$$

where, $E(R_M) - R_f$, is expected excess return on the market portfolio and $\overline{a}$ is the average degree of risk aversion across all investors.

Equation 16 indicates that excess return, or the risk premium, of the market portfolio is proportional to the variance of market portfolio, and the ratio is a measure of the average risk aversion of investors. The left-hand-side of Equation17, which is the ratio of market excess return to market variance, is referred to as the *market price of risk* and plays the crucial role in derivation of the CAPM expected return-beta relationship. In fact, by estimating the expected excess return and the variance of the market portfolio from historical data, one can use Equation17 to estimate the average degree of investors' risk aversion from a historical perspective. Changes in the market risk premium ratio or the market price of risk would, therefore, imply changes in the average investors' risk aversion.

The Expected Return-Beta Relationship

Like Markowitz portfolio selection theory, the CAPM is built upon the assumption that investors evaluate the risk of an individual asset by considering the amount of risk that the asset contributes to the risk of their overall portfolios. And as was shown in the discussion on Markowitz portfolio theory, as the number of securities in a portfolio increases, the effect of the variance of an individual security on total portfolio's variance becomes negligible and the amount of risk that an individual security adds to the portfolio would be the sum of its covariances with other securities in the portfolio. Now, according to CAPM the market portfolio *M* is the only portfolio that all investors invest in. Therefore, the appropriate risk of any individual security is the covariances of that security's return with the returns of all other securities in the market portfolio. By considering the formula for the variance of a linear combination of some variables, it follows that the risk contribution of an individual security *j* to the market portfolio would be:

Risk contribution of security j to the market portfolio M:

$$= 2w_j[w_1 Cov(R_1, R_j) + ... w_k Cov(R_k, R_j.) .... + w_n Cov(R_n, R_j)] = 2Cov(R_j, \Sigma w_k R_k)$$

$$= 2Cov(R_j, R_M) \qquad (18)$$

Therefore, from the perspective of portfolio investment, the risk of any individual security would be twice the covariance of that security's return with the market return. According to Sharpe (1964), equilibrium in the security market requires the price of risk for any security *j* to be equal to the market price of risk. If a security's price of risk is higher than that of the market, then investors will increase their average portfolios' price of risk by increasing the weight of that security in their portfolios until the rise in the price of the security makes its price of risk equal to the market price of risk. This process leads to the following relationship between the return of any individual security ***j*** and the return on the market portfolio:

$$\frac{E(R_j) - R_f}{2Cov(R_{j,} R_M\ )} = \frac{E(R_M) - R_f}{2\sigma_M{}^2}$$

Or:

$$E(R_j) - R_f = \frac{Cov(R_{j,} R_M\ )}{\sigma_M{}^2}[E(R_M) - R_f] \qquad (19)$$

The term $\frac{Cov(R_j, R_M)}{\sigma_M{}^2}$ in Equation19 measures the contribution of security *j* to the risk of the market portfolio measured as a fraction of the variance of the market portfolio and is called *beta* of *j* and is denoted by β_j. Using this notation Equation19 can be written as:

$$E(R_j) - R_f = \beta_j [E(R_M) - R_f] \tag{20}$$

Equation 20 is the well known CAPM expected return-beta relationship. It states that in equilibrium the expected risk premium for any individual security *j* is equal to the expected risk premium of the market portfolio times the beta of the security *j*, where β_j is a measure of the relative risk of security *j* within the market portfolio. Because the β of a security contains only the return covariance of that security with the market portfolio, the β*'s* reflect only the systematic risk of securities; the risk that results from general macroeconomic factors and can not be eliminated through diversification.

The expected return-beta relationship, as expressed in Equation 20 above, has various analytical and practical implications. First, and foremost, because beta is proportional to the risk that a security contributes to the optimal risky portfolio *M*, the expected return-beta relationship shows that there is a direct linear relation between the expected rate of return of a security and its risk. This implies a direct reward-risk relation in any investment, which seems to be a common sense phenomenon. Second, because CAPM concludes that all investors hold a mix of the market portfolio *M* and the risk-free asset, it follows that all investors come up with the same value of β for every specific security and, therefore, for any security the β is a unique and objective measure independent of any individual investor's attitude. Third, as the expected return-beta relationship is true for every security in the market, it can easily be shown that it would also be true for any portfolio made by combining individual assets and the β of the constructed portfolio would be the weighted average of the *β's* of the portfolio's securities, weighted by the respective weights of the securities in the portfolio. Fourth, the computational burden of estimating portfolio's risk is much less than what was required in the Markowitz model. For n securities being traded in the market the Markowitz model required estimating the variance-covariance matrix. This required estimating n variances and $(n^2-n)/2$ covariances, making the total number of estimates to be $n(n+1)/2$. This means that, for example, if there are 500 securities trading in the market one has to estimate 125,250 variances and covariances. However, with the CAPM, estimating the betas of every security, that is, n betas would be sufficient to estimate the systematic risk of any portfolio. Finally, because the covariance of a variable with itself is equal to the variance of the variable, it follows that the β of the market portfolio is equal to *one*. Thus, a security with a β greater than one would be more risky than the market portfolio and a security with a β of less than one would be less risky than the market portfolio.

The Security Market Line (SML)

Sharpe (1964) further develops the CAPM by interpreting the expected return-beta relation expressed in Equation 20 as the relationship between an independent variable, the β, and a dependent variable, the expected return. Thus, Equation 20 can be sketched as a line with y-intercept being the risk free rate and the slope being the risk premium of the market portfolio. This line, as shown in Figure 4 below, was called the *security market line* (SML) by Sharpe (1964). Rearranging the terms in Equation 20, the equation for the security market line can be written as:

$$E(R_j) = R_f + \beta_j[E(R_M) - R_f] \tag{21}$$

Equation 21, being an equilibrium condition, implies that for any security *j* every investor values the security by using the same required rate of return to discount expected future pay-offs and thus all investors come out with the same value for every specific security. And this leads to equilibrium prices of securities in the capital market.

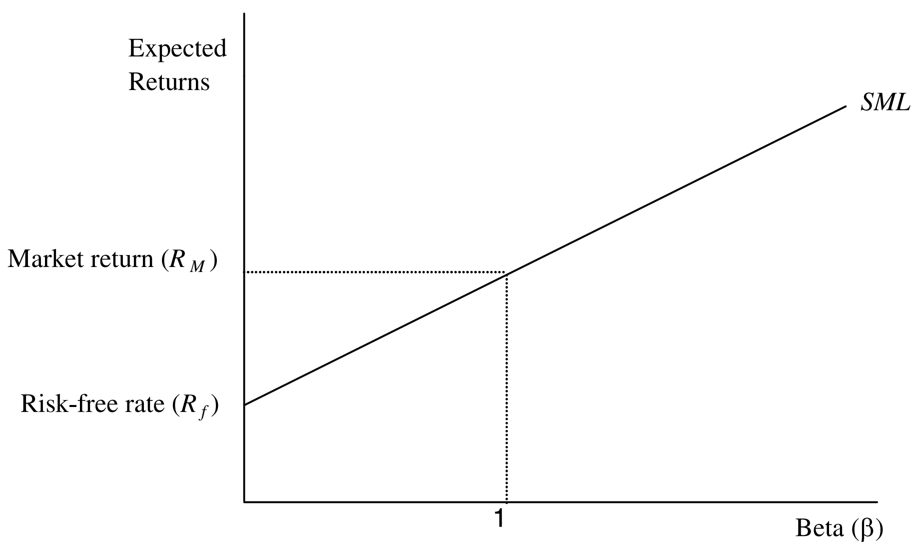

Figure 4. The security market line (SML) and the expected return-beta relationship.

At this point, one has to distinguish the difference between the capital market line and the security market line. The CML expresses risk premium of efficient *portfolios*, consisting of the risk free asset and the market portfolio, as the function of the standard deviation of the portfolio, because standard deviation or the variance of a portfolio is the appropriate measure of the risk of the portfolio. In contrast, the SML relates the risk premium of *individual* securities to their betas, because the beta is the appropriate measure of risk for individual securities held as parts of well-diversified portfolios. However, as was mentioned before, the expected return-beta relation also holds for portfolios of securities and therefore, the SML can be used for portfolios as well as for individual securities. The SML, reflecting the reward-risk relationship of securities at market equilibrium, can be used to evaluate if an individual security is *fairly priced* in the market. Given the beta of a security, if the security is trading at its equilibrium price then its expected return should plot on the SML. If in a particular point of time an estimate of the rate of return of a security obtained *independent* of the CAPM equation falls above the SML, it means that the security is trading below its equilibrium value and vice-versa. The difference between the expected rate of return implied by the SML and the actual rate of return of a security is usually referred to as *alpha* and is denoted by α in the investment management field. This implication of CAPM is used by portfolio managers to try to discover under-valued securities and construct positive alpha portfolios.

Extensions and Modifications of CAPM

There are two classes of extensions and modifications to the original version of CAPM. The first approach tires to relax some of the oversimplified assumptions of CAPM and modify the model to more realistic scenarios. The second approach argues that besides the uncertainty of returns, reflected through the market portfolio, that investors worry about, they also require to be compensated for other sources of risk and thus multifactor CAPMs are proposed.

The Zero-Beta Model

As was discussed earlier, the CAPM assumption of the existence of a risk-free asset that all investors can unconditionally borrow and lend it at the same common risk-free rate leads to the conclusion that all investors arrive at the same optimal portfolio and hold a share of that portfolio. The zero-beta model modifies CAPM by dropping the assumption of the existence of risk-free asset, while keeping all the other assumptions of CAPM intact. Black (1972), who first introduced this version of CAPM, argues that once the inflation uncertainty is accounted for there would be no such thing as a risk-free asset in the economy. Even the US Treasuries that are entirely free of default risk carry inflation risk, because they are nominal obligations and their real values will decline with inflation. Dropping the assumption of the risk-free asset availability or even putting some restrictions on the risk-free asset, such as, different rates of borrowing and lending, leads to the conclusion that the market portfolio would no longer be the optimal portfolio for all investors. Investors may select any portfolio from the efficient frontier depending on the amount of risk they choose to bear. Moreover, with investors choosing different portfolios, it is no longer obvious if the market portfolio, being the aggregate of all investors' portfolios, will even be a mean-variance efficient portfolio. This implies that if the market portfolio is no longer on the efficient frontier, then the expected return-beta relationship of CAPM will no longer reflect capital market equilibrium.

Black (1972) demonstrated, through rigorous mathematical analysis, that theoretically the CAPM findings can be true even when there is no risk-free asset in the economy. Black constructs his model of CAPM in the absence of a risk-free asset by discovering and mathematically proving three properties of the Markowitz's efficient frontier portfolios. These properties are:

1. Any portfolio constructed by combining other efficient frontier portfolios is itself on the efficient frontier. In other words any linear combination of mean-variance efficient portfolios is itself a mean-variance efficient portfolio.

2. The expected return of any asset *j*, whether on the efficient frontier or not, can be expressed as a linear combination of the expected returns of *any two* portfolios on the minimum variance frontier. Specifically Black (1972) proved that for any asset *j* and any two arbitrary portfolios *P* and *Q* on the minimum variance frontier the following relationship always holds:

$$E(R_j) = E(R_Q) + \frac{Cov(R_j, R_P) - Cov(R_P, R_Q)}{\sigma_P^2 - Cov(R_P, R_Q)}[E(R_P) - E(R_Q)] \qquad (22)$$

It should be noted that Equation 22 is a purely mathematical relationship between the expected returns of individual securities and portfolios on the minimum variance frontier and as such has nothing to do with the capital market equilibrium.

3. Every portfolio on the efficient frontier has a *companion* portfolio on the bottom part (the inefficient part) of the minimum variance portfolios with which it is uncorrelated. Because an efficient portfolio and its companion portfolio are uncorrelated, black called the companion portfolio, the *zero-beta* portfolio of the efficient portfolio. Black demonstrated this idea graphically as the following. If *P* is a portfolio on the efficient frontier, then its zero-beta companion, denoted by *Z(M)*, can be derived by first drawing a tangent line on the efficient frontier at point *P*, finding the point of intersection of this tangent line with the vertical axis, and then drawing a horizontal line from this

resulting point. The intersection of this horizontal line with the minimum variance frontier, which is on the inefficient part, will be, *Z(M),* the companion portfolio of portfolio *P*. This idea is shown in Figure 5 below:

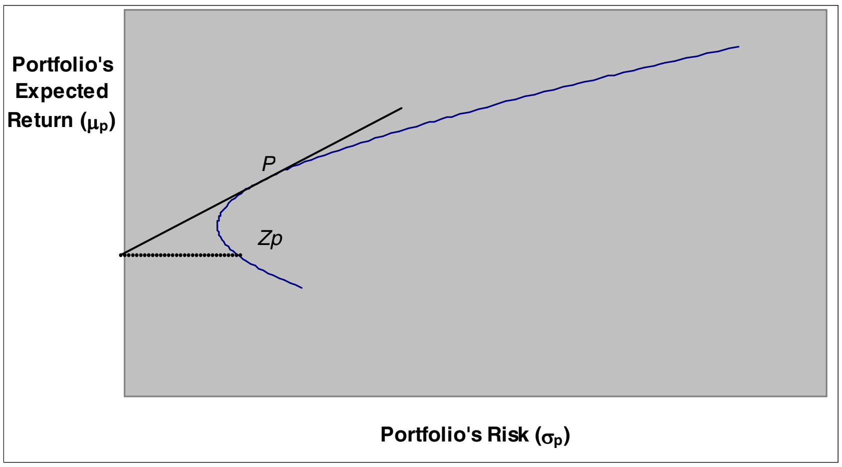

Figure 5. Zero-beta companion of an efficient portfolio.

From these properties of the efficient frontier, Black (1972) employs the following reasoning to drive his model. Like in the standard version of CAPM, the assumption of homogenous expectation implies that all investors use the same inputs and compute the same efficient frontier. Because the market portfolio is the aggregate of all investors' portfolios and because each investor holds a portfolio on the efficient frontier, the market portfolio can be expressed as a linear combination of other efficient portfolios. Therefore, according to property 1 the market portfolio is on the efficient frontier. Again, this conclusion is mathematical and is not a derivation of equilibrium conditions. Now, in Equation 22 of property 3 if instead of choosing two arbitrary portfolios *P* and *Q* from the minimum variance frontier, one chooses the market portfolio *M* and its zero-beta companion *Z(M)* then because $Cov(R_M, R_{Z(M)}) = 0$, Equation 22 will be reduced to:

$$E(R_j) = E(R_{Z(M)}) + \frac{Cov(R_j, R_M)}{\sigma_M^2}[E(R_M) - E(R_{Z(M)})] \quad (23)$$

Equation 23 is just a variant of the expected return-beta relationship of standard CAPM with the difference that the risk-free rate R_f has been replaced with $E(R_{Z(M)})$. However, as can be seen from Equation 23, the beta in Black's model is the same as the beta in the standard CAPM. Thus, Black (1972) proves that the core conclusions of CAPM remain valid even if the assumption of risk-free asset is dropped from the model. Black's model works both when the assumption of the existence of a risk-free asset is dropped and when the risk-free asset does exist but different rates apply to lending and borrowing. Specifically, Black's model demonstrates that (a) the market portfolio is a mean-variance efficient portfolio and lies on the efficient frontier and (b) for every individual security there is an expected return-beta relationship, with the beta being the standardized covariance of the security's return with the market return. The Black model, however, keeps all other assumptions of CAPM. Further attempts by other scholars are done to verify theoretical validity of CAPM if some of the other assumptions are removed.

The CAPM with Taxation

Litzenberger and Rawaswamy (1979, pp. 163-196) developed a CAPM dropping the no-taxation assumption as well as the risk-free assumption. Their model incorporated the effect of different tax rates on dividends and capital gain and restriction on borrowing. The equation they derived for expected return of a stock was similar to the Black's zero-beta model and the standard CAPM, with an additional term whose value depends on the dividend tax rate, capital gain tax rate, and the beta of the stock.

Therefore, they showed that CAPM remains valid even if both risk-free asset assumption and no taxation assumptions are waived.

CAPM and Non-normality of Returns

Kraus and Litzenberger (1976, pp. 1085-1100) argued that return distributions can not be normal, at least for the reason that return possibilities are unlimited on the upside but limited to -100% on the downside, that is, one can not lose more than what invested but might gain any returns. They pointed out that evidence supports the idea that over the long run return distributions are lognormal, rather than normal, which is positively skewed. They assume that investors have preference for positive skewness and thus when making investment decisions consider the skewness as well as mean and variance of returns. They show that dropping the normality assumption does not invalidate the CAPM expected return-beta relationship. In their extended CAPM the expected return of any security becomes equal to its beta times the market return (just like in the standard CAPM) plus its skewness times a zero-beta portfolio. The implication of this extension is that investors' required returns may be a function of the skewness of an asset's returns as well as its beta. This may be significant in the case of highly volatile securities, like technology stocks, with positive skewness, or for the public utilities whose rates are controlled by regulatory authorities causing their future profitability to be constrained on the upside and therefore the returns may be negatively skewed.

CAPM and the Single-Index Model

The single-index model was originally suggested by Sharpe (1963) and ironically although the CAPM was subsequently developed by Sharpe (1964) as a theoretical advance of the single-index model in explaining the capital market pricing, the single-index model has gained more popularity in practice due to the possibility of being tested against measurable observations. The single index model, and more generally the factor models, are based on the observation of the actual security returns and make the least assumptions about the investors' behavior or the working of the capital market. The observation that security returns move in concert, specially the fact that most covariances between security returns tend to be positive, implies that the same economic factors affect the outcomes of many firms. These common factors could be the business cycles, inflation, interest rates, the money supply, oil prices, or others. Thus, unexpected changes in any of these factors will create simultaneous unexpected changes in the rates of return of the entire capital market. Because these common macroeconomic factors are interrelated, the single-index model assumes that all the relevant common factors can be grouped into one macroeconomic indicator and this single factor drives the whole security market. It is further assumed, in the single-index model, that beyond this common effect, all remaining uncertainty in the security returns is company specific. In other words, other than the single common factor there is no other source of correlation between security returns.

With this line of reasoning, the actual or realized rate of return on any specific security *j* will have three components, the part expected or required by the investors as of the beginning of the holding period, the part due to unanticipated changes in the common macroeconomic factors during the holding period, and the part due to unanticipated firm specific events during the period. Mathematically, this idea can be written as:

$$R_j = E(R_j) + M_j + e_j \tag{24}$$

where, R_j is the actual return realized on security j, $E(R_j)$ is the investors' return expectation from security j at the beginning of the holding period, M_j is the effect of unanticipated macroeconomic factors on the return of security j during the holding period, and e_j is the effect of unanticipated firm specific events during the period.

Now, because it is observed that different firms have different sensitivities to macroeconomic factors, M_j can be expressed as the product of the responsiveness of firm j to unanticipated macroeconomic factors and the amount of unanticipated changes in the macroeconomic factors. Therefore, Equation 24 can be written as:

$$R_j = E(R_j) + \beta_j F + e_j \tag{25}$$

where, F is the amount of unanticipated changes in the macro factor and β_j is the responsiveness of security j returns to unanticipated macroeconomic factors. Equation 25 is a *single-factor* model of the security returns in which all the relevant economic factors are summarized into one factor. The above factor model would be of no use without specifying a method to measure the factor F which is assumed to affect every security's return. The single-index model assumes that the rate of return on a broad stock market index, such as that of the S&P 500 Index, can be taken as a proxy for the common economic factor; and that is the reason for the label *single-index* model. With this assumption and measuring returns in terms of their deviations from the risk free rate, Equation 25 can then be converted to the single index model through:

$$R_j - R_f = \alpha_j + \beta_j (R_M - R_f) + e_j \tag{26a}$$

where, α_j is the security's return if the index is neutral, that is if the excess return of the index over the risk free rate is zero, R_M is the realized rate of return of the index, R_f is the rate of return of a risk-free asset such as short-term Treasury Bills, and $\boldsymbol{e}_j$ .is an error term which sums to zero over time.

Taking expectations from Equation 26a above converts the index model to:

$$E(R_j) - R_f = \alpha_j + \beta_j E[(R_M - R_f)] \qquad (26b)$$

If the index portfolio represents the true market portfolio then comparing Equation 2b with the expected return-beta relationship of CAPM, as expressed in Equation 20, shows that both models imply an expected return-beta relationship for every security with the difference that CAPM predicts that the term α_j must be zero for all securities. The reason that in CAPM the term α_j must be zero for all securities is that CAPM is an equilibrium theory of asset pricing and posits that the investors' behavior in holding mean-variance efficient portfolios will price securities such that α_j becomes zero for all securities. However, the index model being a model of realized rather than expected values and making no assumption of market equilibrium, allows α_j being positive or negative as well as zero. It then becomes a matter of empirical observation to verify if the average α_j for an individual security over a long period of time becomes zero or not.

Equation 26a of the single-index model implies that for each security there are two sources of risk; *market* or *systematic* risk attributable to the security's sensitivity to macroeconomic factors (reflected in R_M), and the *nonsystematic* or *firm-specific* risk as reflected in the e_j. This idea can be formulated mathematically by applying the formula for the variance of sum of two or more variables. As the firm-specific element of return is uncorrelated with market return and as the variance of a constant value is zero, taking second moments from both sides of Equation 26a yields:

$$\sigma_j^2 = \beta_j^2 \sigma_M^2 + \sigma^2(e_j) \tag{27}$$

where, σ^2 stands for variance of returns. Therefore, total return variance, or total risk, of a security is composed of the systematic part which is equal to the product of the square of its beta and the market return variance, $\beta_j^2 \sigma_M^2$, and the firm-specific variance which is $\sigma^2(e_j)$. Furthermore, by writing Equation 16a for two different securities, considering that the firm- specific components of two securities are uncorrelated, and applying the covariance formula it is deduced that:

$$Cov(R_i, R_j) = Cov(\beta_i R_M, \beta_j R_M) = \beta_i \beta_j \sigma_M^2 \tag{28}$$

Specifically, if one of the portfolios in Equation 28, say portfolio *i*, is taken to be the market portfolio then the beta of any individual security *j* can be expressed as its return covariance with the market return divided by the variance of the market return. That is:

$$\beta_j = \frac{Cov(R_M, R_j)}{\sigma_M^2} \tag{29}$$

Thus, the index model beta coefficient turns out to be exactly the same beta as that of the CAPM expected return-beta relationship with the difference that the theoretical market portfolio of CAPM is replaced with a well-specified and observable index.

Equation 28 implies that the covariance between the returns of two different securities is related to the responsive of each security's return to the market return and the variance of the market return. Putting it simply, it means that correlations between security returns come from a single factor; the market index factor which is the proxy for the common economic factors. For practical purposes using Equations 26-29 one can estimate the input list for the whole universe of securities by first estimating the expected market return and the betas of each security and then use them to estimate the variances and covariances. This means that in the capital market with *n* securities trading in it, *(n+1)* initial estimates enables one to estimate the inputs that are required for constructing any portfolio. This facility of the single-index model, however, is not without cost. The assumption of the single-index model that uncertainty of returns comes only from two sources, the macro and the micro sources, overlooks the fact that while certain macro factors affect all securities, there are some factors that affect many firms within the same industry without substantially affecting the broad macro economy. For example, the price of a raw material used within the computer industry affects all firms within the computer industry without much effect on the firms in other industries. Even those common macro factors that affect all firms in the economy, the magnitude of their effect is different on firms of different industries and thus aggregating all common factors into a single factor could overestimate or underestimate these differences. For example, firms in the public utility and banking industries show different responsiveness to a sudden change in interest rate than do the firms in the technology industry.

The return-beta and other relationships of the single-index model apply to portfolios of securities as well as to individual securities. If Equation 26a is written for every security in a portfolio *p* and the results on both sides of the equations are weighed by the weights of individual securities and then added up, the following relationship follows:

$$R_p - R_f = \alpha_p + \beta_p(R_M - R_f) + e_p \tag{30}$$

where, the return, the beta, and the portfolio-specific return component of return are equal to the weighed average of returns, betas, and firm-specific returns of individual securities, respectively. And from Equation 30 it follows that:

$$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma^2(e_p) \tag{31}$$

Equation 31 provides significant insights on the benefit of portfolio diversification. It implies that if the number of securities in a portfolio is increased, the firm-specific component of the portfolio's risk, $\sigma^2(e_p)$, will tend to zero and will be diversified away. This is usually proved, mathematically, by considering a portfolio of *n* securities with equal weight of *1/n* for each security, in which case the nonsystematic component of the portfolio's risk would be:

$$\sigma^2(e_p) = \frac{\sum \sigma^2(e_j)}{n^2} = \frac{\bar{\sigma}^2(e)}{n} \tag{32}$$

where, $\bar{\sigma}^2(e)$ is the average of firm-specific return variances in the portfolio. Equation 32 shows that the more the number of securities in a portfolio the smaller would be the specific risk of the portfolio and therefore, in a sufficiently diversified portfolio the specific risk tends to zero and only the systematic risk $\beta_p^2 \sigma_M^2$ remains which is not diversifiable. This idea of diversification benefit is shown graphically in Figure 6 below:

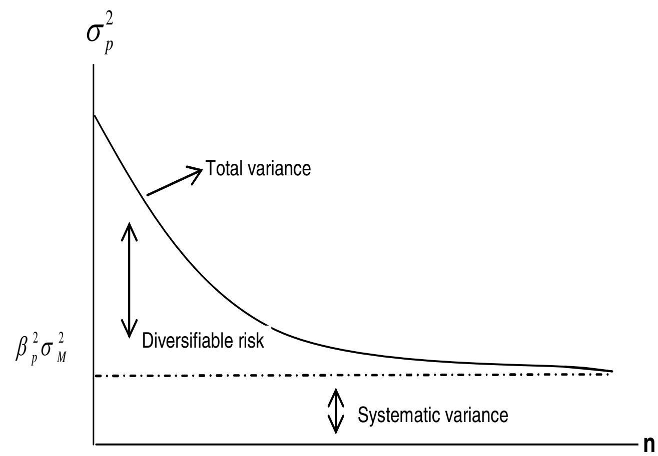

Figure 6. Diversification and risk reduction.

The advantage of the single index model over the CAPM is that it arrives at similar return-risk relationship while overcoming the testability problem of CAPM. As was discussed before one central tenet or prediction of CAPM was that the market portfolio is a mean-variance efficient portfolio. To test for this, one needs to build a value-weighted portfolio of enormous size that contains all the assets in the market. Such a task has not been done so far, if at all feasible, and all the tests conducted use a much smaller portfolio as a proxy for the market portfolio. Moreover, even if the tests indicate that the market proxy is an efficient portfolio, as Ross (1977, pp. 129-176) points out in his famous critique of the CAPM, this does not prove that the actual market portfolio is efficient and thus can not testify CAPM tenet of the efficiency of the market portfolio. An even more serious problem with CAPM is that it implies relationships among *expected returns*, whereas what can be observed in reality are actual or realized holding period returns that are not necessarily equal to the prior expectations. In fact this problem of *ex-ante* versus *ex-post* returns makes it impossible to test the CAPM even if an actual market portfolio is constructed. To test for the efficiency of the market portfolio, one needs to show that the reward-to-variability ratio of the market portfolio is higher than that of all other portfolios. But the reward-to-variability in CAPM is set in terms of expectations, and there is no way to observe expectations directly. This problem of measuring expectations becomes troublesome for testing the second central set of CAPM predictions, those of the expected return-beta relationship of individual securities and the resulting security market line. These relationships are expressed in terms of the expected returns of individual securities and the expected return of the market portfolio and thus one need to measure the unobservable expectations for testing them. Because of these two major problems that the market portfolio and the expected returns are not observable in the CAPM, the betas of individual securities can not be empirically determined by CAPM and therefore the expected return-beta relationship for the universe of securities as exemplified in the SML can not be tested.

The single index model overcomes these shortcomings evidenced in the empirical testing of CAPM by using (a) a broad index as a proxy for the market portfolio and (b) the average historical rates of returns of the index and of individual securities as the proxies for the expected rates of return. The first step in the single-index model is to estimate the beta of each security through time-series regression analysis. If for an individual security *j*, the index model formula is written for every holding period over a long period of time, then the index model's return-beta relationship for security *j* will become:

$$R_{tj} - R_{tf} = \alpha_j + \beta_j (R_{tM} - R_{tf}) + e_{tj} \text{, for } t = 1, 2, \ldots, k \qquad (33)$$

Treating excess return for the index as the *independent* variable and the excess return for security *j* as the *dependent* variable in Equation 33 and solving for the regression coefficients provides estimates for the *y* intercept α_j and the slope β_j of the regression line. Therefore, for every security ***j*****,** the index model estimates a line of the form:

$$R_j - R_f = a_j + b_j(R_M - R_{tf}) \tag{34}$$

where, a_j and b_j are the regression estimates for the true coefficients α_j and β_j respectively. The line obtained from Equation 34 above is called the *security characteristic line* (SCL) for security ***j*****.**

Empirical Tests of CAPM

Despite the wide controversy about the validity of various conclusions and implications of CAPM, the financial community has issued favorable judgment on the CAPM, basic version or modified versions, and is utilizing the implications of CAPM in widely varying applications. Major application of CAPM for practical financial decision making include:

1. Professional portfolio managers use the expected return-beta relationship of securities to detect undervalued or overvalued securities to make buy-sell decisions.

2. Mutual fund companies and institutional investors judge performance of their portfolio managers by using the reward-to-variability ratio they realize or the average rates of returns they achieve as compared to the SML.

3. Regulatory commissions use the expected return-beta relationship along with forecasts of the market index return as one factor in determining the cost of capital for regulated firms.

4. Court rulings on damage claim cases sometimes use the expected return-beta relationship to determine discount rates to evaluate claims of lost future income.

5. Many firms use the SML to obtain a benchmark cost of capital to use for their capital budgeting decisions (Patterson, 1995, pp. 170-185).

These considerations show that the idea of risk-return relationship is generally accepted as a normative or as a positive proposition. Therefore, while reviewing the empirical tests of the CAPM conducted by others, it is pertinent to evaluate the results with the view of developing or modifying the model along the lines more compatible with new observations: and this is what will be done in this research.

Because neither the theoretical market portfolio nor return expectations are observable, all tests of CAPM typically involve some variety of the index model. Early simple tests followed three basic steps (a) establishing sample data by specifying the sample time span, the length of holding period, the index to be used , and the securities to be included for the study, (b) estimating the security characteristic line (SCL) for every security in the sample, and (c) estimating the security market line (SML). With *k* holding periods within the sample time span, the sample data would consists of (a) *k* holding period rate of returns, R_{tj} for each security *j* during the sample time span, (b) *k* holding period rate of returns on the index, R_{tM} , during the sample time span, and (c) *k* holding period risk free rates, R_{tf} for every holding period during the sample time span. The sample data are then used as inputs for a *two-pass regression*, such that the estimates obtained from the first regression equation are used as data inputs for the second regression equation. In the *first-pass regression,* for every security in the sample the excess return of the security is treated as the dependent variable and the excess return of the index is regarded as the independent variable. The slopes of the resulting regression lines (SCLs) provide estimates for the betas of every security in the sample. Thus, the data for the first-pass regression are time-series data collected for the variables of the model. Form the first-pass regression model and the sample data the following information are then used for the *second-pass regression*:

1. $\overline{R_j - R_f}$ = Sample averages of every security's excess holding period return over the sample period, representing security's expected holding period return.

2. $\overline{R_M - R_f}$ = Sample average of excess return of the index, representing the index's expected holding period return.

3. b_j = Regression estimates of the beta coefficient of each of the securities in the sample.

4. $\sigma^2(e_j)$ = Estimates of the variance of the residuals for each of the securities in the sample.

In the second-pass regression, the beta estimated from the first-pass regression is regarded as the independent variable and the security average excess return over the sample period is viewed as the dependent variable. With *n* securities in the sample there are now *n* estimates for the betas, one for each security, and *n* average excess returns, one for each security. The data for the second-pass regression are therefore cross-sectional. The second-pass regression equation, with b_j and $\sigma^2(e_j)$ estimated from the first-pass regression, is then formulated as per Equation 35 below:

$$\overline{R_j - R_f} = \lambda_0 + \lambda_1 b_j + \lambda_2 \sigma^2(e_j) \tag{35}$$

Because CAPM proposes that expected excess return of a security is equal to the beta of the security multiplied by the expected excess return of the market, to test the CAPM propositions Equation 35 is estimated against the following hypotheses:

$$\lambda_0 = 0,\ \lambda_1 = \overline{R_M - R_f},\ \text{and}\ \lambda_2 = 0$$

If the above hypotheses are not rejected by the regression analysis, then Equation 35 will provide an estimate of the security market line, the SML, in conformity with CAPM predictions.

The first comprehensive empirical test of CAPM following above pattern was conducted by Linter (1965, pp. 13-37) who used annual data on 631 New York Stock Exchange (NYSE) stocks for 10 years, 1954 to 1963, and obtained the following results as shown in Table 1 for the regression coefficients:

Table 1

Linter's Results for Test of the CAPM

	λ_0	λ_1	λ_2
Coefficient	12.70%	4.20%	31%
Standard Error	1%	0.60%	2.60%
Sample average for annual excess market return, $\overline{R_M - R_f} = 16.5\%$			

Linter's findings are totally inconsistent with the CAPM hypothesis. First, the estimated SML is too flat, that is, the λ_1 coefficient was too small. The slope should be equal to 16.5% which is the sample average for excess market return, but it was estimated to be 4.2%. The difference, 12.3%, is about 20 times (t = *20.5*) the standard error of the estimate, which rejects the null hypothesis that the slope is 16.5%. Also the intercept of the SLM, λ_0, hypothesized to be zero, is estimated to be 12.7%, which is more than 20 times (t = *21.16*) the standard error of 0.6% and thus the null hypothesis that the intercept is zero is rejected. Second, and more destructive to the CAPM, is that nonsystematic risk seems to predict expected excess returns of securities. The coefficient, λ_2, which measures the contribution of nonsystematic risk, $\sigma^2(e_j)$, to expected returns, is 31%, more than 10 times its standard error of 2.6%, leading to the rejection of the null hypothesis that nonsystematic risk does not affect stock returns.

Proponents of the CAPM interpreted the disappointing findings of Linter's test to be due to two specific problems. First, the beta estimates obtained from the first-pass regressions involve *measurement error*. Using these estimates in place of the true beta coefficients in the estimation of the second-pass regression biases the estimates and leads to an estimate of the SML that is too flat and has a positive rather than zero intercept. Second, it is noted that in the first-pass regression the variance of the residuals shows statistically significant correlation with the beta coefficients of the stocks. Stocks that have high betas tend to have high nonsystematic risk. This effect together with the effect of measurement error leads to an upward bias in estimation of the coefficient of nonsystematic risk, λ_2, in the second-pass regression (Patterson, 1995, pp. 53-70).

Therefore, the next wave of tests was designed to overcome the problem of the measurement error and that of nonsystematic risk which together led to biased estimates of the SML. The innovation in these tests was to use portfolios rather than individual securities. Combining securities into portfolios reduces the nonsystematic risk and diversifies away most of the firm-specific portion of the returns. Although combining stocks into portfolios reduces the number of observations left for the second-pass regression but it increases the precision of the estimates of the beta coefficients in the firs-pass regression and thereby it mitigates the statistical problems that arise from measurement error in the beta estimates. Test of the CAPM along this line of thinking was pioneered by Black, Jensen, and Scholes (1972, pp. 79-121). They tested the prediction of CAPM by investigating the empirical expected return-beta relationships for portfolios of stocks using all stocks trading on NYSE for the period 1926 through 1965. To construct portfolios suitable for testing the expected return-beta relation of CAPM, Black, Jensen, and Scholes (BJS) first estimated the betas of all stocks in the sample for the subperiod 1926-1930, by regressing monthly excess return of every security against the excess return of an equally weighed portfolio of all the stocks in the sample and ranked the stocks based on those estimates of the betas from the highest to the lowest. They then partitioned the sample into ten portfolios starting from the stock with the highest beta and ending with the stock with the lowest beta. Subsequently, BJS conducted the first-pass regression by calculating the monthly excess returns of the ten portfolios for the period 1931-1965 and regressed them against the monthly excess return of the market to get beta estimates for the portfolios. Finally, the ten observations on the average excess return of the portfolios were regressed against the portfolios' beta estimates in the second-pass regression to obtain an estimate of the SML. Because of dealing with portfolios, rather than individual stocks, BJS used a two parameter CAPM for their second-pass regression, that is, they let $\lambda_2 = 0$ in Equation 35 above. Their findings are summarized in Table 2 below:

Table 2

BJS Results for Test of the CAPM.

	λ_0	λ_1
Coefficient	0.359%	1.080%
Standard Error	0.055%	0.052%
Sample average for monthly excess market return, $\overline{R_M - R_f} = 1.42\%$		
R^2 =98%		

The BJS results do not seem to provide much support for the CAPM hypothesis either. The intercept of SML, λ_0, is equal to 0.359% which is more than 6 times its standard error (*t = 6.53*), and thus the intercept which is hypothesized by CAPM to be zero is positive and statistically significant. For the sample period, the average monthly excess return on the market was 1.42% which according to CAPM should be equal to the slope of SML. But the slope coefficient, λ_1, was estimated to be 1.08%, lower than the hypothesized value by 0.34%. This difference is more than 6 times the standard error of the slope estimate (*t = 6.18*), which is statistically significant and thus leads to the rejection of the null hypothesis that the slope of SLM is equal to average excess market return. Although the BJS test results were negative for the CAPM hypotheses, but the high R-square of 98% indicated the existence of high positive correlation between excess portfolio return and portfolio beta. However, BJS, did not concern themselves with other implications of CAPM, such as that expected returns are independent of nonsystematic risk or that the relationship between expected returns and beta is linear.

To address the shortcomings of the BJS test, Fama and MacBeth (1973. pp. 607-636), used the BJS methodology to verify that the observed relationship between portfolios' average excess return and beta is indeed linear and that nonsystematic risk does not have significant effect on portfolio's excess return. Thus, they expanded the equation for estimation of SML to include the square of the beta coefficient and the estimated standard deviation of the residual term. They used the same database as the BJS, but divided the sample into 20 portfolios, containing securities from the highest to the lowest beta and after estimating the beta of the portfolios in the first-pass regression for the period 1930-1965, they used the beta estimates to test the following expanded SML equation:

$$\overline{R}_{p,t} = \lambda_0 + \lambda_1 b_{p,t-1} + \lambda_2 b^2_{p,t-1} + \lambda_3 \sigma(e_{p,t-1}) \tag{36}$$

where, the term λ_2 measures the potential nonlinearity of returns and the term λ_3 measures the explanatory effect of the nonsystematic risk. Also Equation 36 relates average monthly returns, instead of excess average monthly returns, of portfolios to the independent variables with a one-month lag period. Thus, the model is designed to test the CAPM through the following hypotheses: $\lambda_0 = \overline{R_f}$, $\lambda_1 = \overline{R_M - R_f}$, $\lambda_2 = 0$, and $\lambda_3 = 0$, where $\overline{R_f}$ is the average monthly risk free rate for the sample period. Famma and MacBeth results are shown in Table 3 below:

Table 3

Fama and MacBeth Results for Test of the CAPM

	λ_0	λ_1	λ_2	λ_3
Coefficient	0.20%	1.14%	-0.26%	5.16%
t-statistic	0.55	1.85	-0.86	1.11

Sample average for monthly excess market return, $\overline{R_M - R_f} = 1.30\%$
Sample average for monthly risk-free rate, $\overline{R_f} = 0.13\%$

Fama and MacBeth results are more in support of a two-parameter CAPM than the previous tests. The reported *t* statistic indicate that λ_2 and λ_3 are not significantly different from zero, λ_0 is not significantly different from $\overline{R_f}$, and that given the large sample size λ_1 is not significantly different from the average monthly market excess return of 1.30%. However, the estimated SML is still too flat as compared to the expected theoretical SML, that is, an estimated slope of 1.14% in contrast to the expected slope of 1.30% for the SML.

Contemporary tests of CAPM follow two distinct lines of thinking. The first, pioneered by Fama and French (1992) seeks to determine other factors, in addition to the CAPM beta, that could explain variations in rate of returns in cross section of stocks. The second line of thinking, initiated by Jagannathan and Zhenyu.(1996) is based on the idea that the CAPM assumption of single-holding period implies that the risk factor, beta, is constant over time, while empirical evidence suggest that investors change their attitudes towards risk as economic conditions change.

Fama and French (1992, pp. 427-464) developed a three-factor model using a similar method to that employed by Fama and MacBeth (1973) and tested the model for the sample period 1941-1990. But, their findings are less supportive of the CAPM. They found that the simple relationship between beta and average return disappears during the more recent 1963-1990 period and that even for the expanded period of 1941-1990 the relationship between beta and average return is also weak. Instead, Fama and French (1992) found that two other variables, size (as measured by the market value of the equity) and book value divided by the market value of the equity provide strong characterization of the cross-section of average stock returns for the 1963-1990 period. The authors conjectured that because of the relationships between these variables and systematic patterns in relative profitability and growth across firms, these variables can be viewed as proxies for the common risk factors for all securities.

In a follow-up paper, Fama and French (1995, pp. 131-155) explored the time-series relationship between asset returns and returns on portfolios constructed to mimic the risk factors related to size and book/market ratios. They reported that size and book-to-market equity proxy for sensitivity to common risk factors in stock returns. However, they also indicated that although these factors explained differences in average returns across stocks, but still the return on the market portfolio was required as a third variable to explain the large difference between the average return s on stocks and the returns on the Treasury Bills.

Jagannathan and Wang (1996, pp. 3-53) developed and tested a model of time-varying β, known as conditional CAPM. They argued that the reason that Fama and French (1992) found β to be insignificant in their study is because of changing investors' attitude towards risk due to changes in economic conditions during their sample period. Thus, Jagannathan and Wang construct their conditional expected return model by allowing β and the expected market risk premium to be conditional upon some common information known to the investors prior to each specific time period. However, they do not present any theoretical guidance as to how to distinguish between the specific time periods for which investors' attitudes towards risk changes. Instead they decompose their proposed conditional β into an unconditional market β_{vw} and a new variable which is also time unconditional and they call it market risk premium β_{prem}. To test their model Jagannathan and Wang (1996, pp. 3-53), choose the yield spread between AAA and BAA rated corporate bonds as the proxy for the market risk premium and their value weighted stock index as proxy for the market. They then regress realized rate of returns of securities against these two variables and estimate the market risk premium β_{prem}, and the market β_{vw}.for their sample of securities. They also calculate separate estimate for β_{vw} without including the β_{prem} in the model. Jagannathan and Wang (1996) found that including the market risk premium variable to reflect changing economic conditions improves the explanatory power of the model over the static model in which the market index was the only independent variable. Their conclusion is that the expected return-beta relationship of CAPM is valid if some proxy for change of investors' attitude to risk is included in the model.

To sum up, tests of CAPM using realized or *ex post* returns as proxies for the expected or *ex ante* returns do not lead to consensually unambiguous conclusions with respect to the expected return-beta relationship as posited by the CAPM. And the results of such tests are dependent on the periods used and the technique employed. But the model has been employed as a useful analytical tool and has survived as a practical aid to financial decision maker over the past 30 years. It is therefore worth while to extend the model along some lines more consistent with empirical observations while preserving the basic ideas of the model, especially the intuitively appealing view that return and risk are related. In this endeavor the author of this dissertation suggests to (a) look for some other proxies for the ex-ante returns, in particular for the market portfolio, (b) in combination with beta include other possible omitted variables, that might explain differences in returns across securities, (c) empirically verify if the beta of securities remain constant over time, and if beta changes over time then modify the CAPM for time-varying beta, and (d) empirically investigate whether variance could be a better risk index than the beta given the statistical significance of the nonsystematic risk reported by previous tests.

The Arbitrage Pricing Theory

The arbitrage pricing theory (APT) was first proposed by Stephen Ross in 1976 as an alternative model of asset pricing in the capital market challenging the CAPM assumptions and conclusions. The APT model is based on the notion of *arbitrage* in economic activity and the role of arbitrage in maintaining equilibrium in the markets. In economic theory, arbitrage is defined as the possibility of making a *sure* profit with *zero* investment and without taking any risk. This possibility arises when the *law of one price* is violated; meaning that an item trades for different prices in different markets at the same time or if the item is mispriced in the market. If such an arbitrage opportunity comes up in the capital market, then the arbitrageur will construct a risk-free portfolio by selling short the overpriced security (or borrow at the risk-free rate which is the same thing as selling short the risk-free asset) and buying the underpriced security with the proceed, thus, make a sure profit with no owned investment. The critical implication of this risk-free arbitrage portfolio is that any investor, regardless of risk aversion or wealth, will want to take an infinite position in it and make infinite profits. Because of these large intended positions, it takes only a few arbitrageurs to force security prices to move up and down until the arbitrage opportunity is vanished and equilibrium in the market is restored. This is the core difference between APT explanation of equilibrium asset prices and that of CAPM. The CAPM argues that *all* or a sufficiently large number of investors hold mean-variance efficient portfolios. If some securities are mispriced in the market, in the sense that their expected returns do not reflect their expected risks as stipulated in *Equation 22* above, then a large number of investors will tilt their portfolios away from the overpriced securities and shift toward the underpriced ones. Price equilibrium in the CAPM is the result of small shifts in the portfolios of many mean-variance efficient investors. In contrast, the APT model makes no assumption about investors being mean-variance portfolio holders and it does not require the actions of many investors to restore equilibrium prices in the capital market. The no-arbitrage condition of APT states that even relatively few investors are enough to identify an arbitrage opportunity, mobilize large dollar amounts to take advantage of it, and exert pressure on the prices to restore equilibrium (Cochrane, 2001, pp. 63-75).

According to APT model any deviation from equilibrium prices in the capital market is short-term and the working of arbitrage ensures return to equilibrium. But what are those equilibrium prices and how are they determined? The answer to this is similar to that of the CAPM, that is, for every security investors expect a specific rate of return from the security and then discount the expected pay-off from the security at that expected or required rate of return to arrive at the equilibrium price of the security. So, like in CAPM, the problem of equilibrium price determination in the APT model leads to the problem of how investors from expectations with regard to the rate of return that they require from securities.

Like the CAPM, the APT model is an expected return-beta relationship model. But unlike CAPM that condenses all systematic risks of securities into only one source, the *unobservable* market portfolio, the APT argues that any number of macroeconomic factors could be the common source for systematic risks for all securities' return. In this way, the APT model overcomes a major practical problem of CAPM, that of finding a suitable proxy for the market portfolio. Thus, any index that CAPM employs as a proxy for the market portfolio can be incorporated, if needed, as one of the relevant factors in the APT model without requiring the index to be a mean-variance efficient portfolio. The common macroeconomic factors in APT model that affect all securities could be the growth of GNP, inflation rate, exchange rates, interest rate spreads, and so on. Each security shows a different response to these factors and therefore for each security investors require distinct compensations for being exposed to each of these factors. Obviously not all security prices are affected similarly by the factors envisaged in the APT model. When interest rates change, the prices of interest-sensitive stocks, like stocks of banks or utility companies, are affected more than the stocks of technology companies. Or, when the rate of growth of GDP slows, the stock of cyclical companies, like consumer durables or hotels, are more severely affected than the stock of drug companies. The APT model incorporates these response differences of different securities by defining distinct beta coefficients for each security with regard to every common factor. Therefore, the APT model expresses R_{it}, the return on security i at time t, to be:

$$R_{it} = E\,(R_i) + \beta_{i1}F_{1t} + \beta_{i2}F_{2t} + \ldots\ldots \qquad \ldots. + \beta_{ik}F_{kt} + e_{jt} \qquad (37)$$

where, F_{kt} is the deviation of the kth factor from its mean or expected value at time t, β_{ik} is the sensitivity of R_i to this deviation, and e_{it} is a mean-zero random term which is asset specific and unrelated to any of the common factors and therefore its effect can be eliminated through diversification. An implication of Equation 37 is that the expected return on a security is a linear function of the risk free rate and the risk premiums related to each of the common factors that the investors require to be compensated in order to invest in a security. This is expressed as:

$$E\,(R_i) = E_0 + \beta_{i1}E_1 + \beta_{i2}E_2 + \ldots \qquad \ldots. + \beta_{ik}E_k \qquad (38)$$

where, E_0 is the nominal return on a risk-free asset and E_k is the excess return over E_0 in a portfolio with unit sensitivity to factor k and zero sensitivity to all other factors. Therefore, $\beta_{ik}E_k$ would be the risk premium related to factor k that investors require in order to invest in security i. The relationship between the actual rate of return and the expected rate of return of security i at time t would then be:

$$R_{it} = E\,(R_i) + e_{it} \qquad (39)$$

To put it in a simple language, what Equations 39 and 38, and in general the APT model, are saying is that to invest in a security an investor requires to be compensated for all risks arising from the common factors affecting security returns. The risks not related to common factors are specific to the security and can be diversified away and thus the investors do not seek compensation for them. Moreover, if actual return of a security deviates from its expected return the process of arbitrage will quickly eliminate this deviation by putting pressure on the security's market price and thereby brings the actual return in line with the expected return (Reilly & Brown, 1997, pp. 297-304).

The APT model is difficult to assess empirically because it does not specify the identity or the number of factors to be included in the model. All that the theory provides is a structure of asset pricing; it does not specify the common economic factors or the company-specific characteristic that determine rates of return of assets.

Summary

Modern investment theory started with the Markowitz (1952) paper on portfolio selection theory. Markowitz demonstrated that as long as the rates of return of securities are not perfectly positively correlated, portfolio's risk can be reduced through diversification. Furthermore, Markowitz proved, mathematically, that out of all feasible portfolios that can be constructed from the universe of risky assets, there are some portfolios that embody minimum risk for a specific level of expected return, and the set of all such portfolios form a hyperbola called minimum variance frontier. The upper part of minimum variance frontier represents portfolios that have the highest expected returns for the same level of risk and is called the mean variance efficient portfolios or the efficient frontier. Markowitz posits that all rational investors select or should select portfolios from the efficient frontier depending on their degree of risk aversion.

Although Markowitz portfolio selection is not by itself a theory of equilibrium asset prices, it forms the basis of the CAPM developed by Sharpe (1964), Linter (1965), and Mossin (1966). The CAPM extends Markowitz model by introducing the risk-free asset which investors can combine it with a portfolio on the efficient frontier through lending and borrowing it at the fixed risk-free rate. With this addition and by positing that rational investors would select portfolios that yield highest expected return in excess of the risk-free rate per unit of risk taken, CAPM concludes that there exists one specific portfolio on the efficient frontier that could serve as the optimal portfolio for all investors. That single optimal portfolio consists of all risky assets weighted by their market capitalization and labeled the market portfolio. From here, the famous expected return-beta relationship of CAPM is deduced according to which the expected excess return of any security is linearly related to the beta of the security and the expected excess return on the market portfolio. The expected return-beta relationship is the basis of equilibrium asset prices in the capital market and if any deviation from equilibrium prices occur in the market the mean variance efficient investors acting on the basis of expected return-beta relationship will readjust their portfolios and restore equilibrium prices.

The APT was developed by Ross in 1976 to explain the problem of capital market asset pricing without the need to make many of the assumptions of CAPM and without the need to identify a proxy for the market portfolio. The APT is a multi-factor expected return-beta model in which the expected return of any security is a linear function of risk premiums of various factors and the security's return responsiveness to those factors. According to APT model if asset prices deviate from what is implied by their expected return-beta relationships, then large portfolio adjustment of a few arbitrageurs will restore prices to their equilibrium levels.

CHAPTER 3:

METHODOLOGY AND RESEARCH DESIGN

Introduction

Chapter 3 deals with the research design and the methodology employed in this study. In the first section, an overview of the type of research design employed, the nature of the relationships to be studied, and the specific research hypothesis to be investigated are presented. The second section deals with describing the target population, type of data, sources of data, sampling frame, sampling design, and the pilot study conducted to determine the appropriate sample size. The final section pertains to data analysis, which includes detailed description of the variables of the model and their operational definitions, the hypotheses to be tested, and the type of statistical test to be employed.

Research Design and Approach

This study is explanatory, quantitative, and uses available data to investigate and test relationships amongst variables. The dependent variable in the study is the *expected rate of return* on stock of public companies and the independent variables are various risk factors for which investors in the stock require compensations. The independent variables, the risk factors, considered for this study are of two types (a) macro, or systematic, variables and (b) micro, or nonsystematic (company-specific) variables. The macro, or systematic, risk factors are the expected rate of return on the overall stock market and the implied volatility of the overall stock market. The micro, or company-specific, variables are financial leverage, operating leverage, and asset size of the company. These variables were defined in chapter 1, but will be defined again in the data analysis section of this chapter. In this study the relationships between the dependent variable and the independent variables are examined by employing *causal- comparative* research design in order to investigate correlations and regressions. The reason for choosing the causal-comparative research design is because the independent variables of the model can not be experimentally manipulated and, therefore, it is not possible to collect data through experimentation and employ experimental designs. The causal-comparative research design using existing data is the only appropriate design for this study.

The model developed is based on five hypotheses, which are all related to the relationships between rates of return on stocks of public companies and relevant risk factors. The first hypothesis is to test the validity of standard CAPM for the whole sample period January 1, 1995 to Dec 31, 2004. Hypotheses two, three, and four are this author's propositions to bridge the field of investment theory to the field of corporate finance theory. And hypothesis five is this author's proposed contribution to the field, which is stated in terms of the following two regression equations:

$$R_{jt} - R_{ft} = \alpha_j + \beta_j^M (R_{tM} - R_{ft}) + \beta_j^{IV} (IV_{t-1}) + \beta_j^S (LLS_t) + \beta_j^{FL} (HFLLF_t) + \beta_j^{OL} (HOLLO_t) + e_{jt}$$

$$\overline{R_j - R_f} = \lambda_0 + \lambda_1 b_j^M + \lambda_2 b_j^{IV} + \lambda_3 b_j^S + \lambda_4 b_j^{FL} + \lambda_5 b_j^{OL} + e_j$$

where, the β_j's are the responsiveness of stock *j* rate of return to the five risk factors specified above and the b_j's are estimates of the β_j's found from the first regression equation.

Population and Sampling Frame

The target population of the study entails all publicly traded companies in the U.S. whose securities are traded in New York Stock Exchange (NYSE) and or on NASDAQ National Market. Therefore, the study does not contain private companies or companies with stocks trading on the Pink Sheet or OTC Bulletin Board due to nonavailability of adequate financial data on these companies. The sampling frame chosen for this target population is the list of companies whose stocks constitute the Russell 3000 Stock Index. The reason for choosing this sampling frame is that, unlike other stock indices, the Russell 3000 Stock Index consists of only the stocks of U.S. companies and as such it does not include stocks of non-U.S. companies trading in the U.S. stock market or the American Depository Receipts (ADRs) which represent stocks of non-U.S. public companies. Moreover the 3000 stocks that make up the Russell 3000 Stock Index represent approximately 98% of the U.S. equity market (http://www.russell.com). The list of companies that constitute Russell 3000 Stock Index was downloaded from Russell Company's website and was used as the sampling frame for this study. However, because this study covers the period January 1995 to December 2004, the sampling frame was truncated to include only those companies that became public prior to the year 1995 and for which the required data were available for the study period.

The unit of analysis, therefore, is each company in the Russell 3000 Stock Index that has been operating as public company for the whole sample period of January 1995 to December 2004 and for which price and financial data was available for the study period. Characteristics, or variables, of the units of analysis that will be studied are monthly closing stock prices, variability of monthly stock prices, dividends paid by the company to common stockholders, and the company's annual balance sheets from which the items total assets, fixed assets, and long-term debt are retrieved to measure the company specific variables size, financial leverage, and operating leverage.

Sampling Design

Sampling design in this research is stratified random sampling. The criteria for stratification are company's asset size, company's financial leverage, and company's operating leverage. For asset size, companies have been grouped into large size (LAS) and small size (SAS) companies. The procedure for do this grouping is as follows (a) first, for every company in the sampling frame the average asset size for the study period was calculated by summing up the company's asset value in its balance sheet as of Dec, 31, 1994 and as of Dec, 31, 2004 and dividing the result by two, (b) second, companies were ordered based on their average asset size during the study period and the median asset size was calculated, and (c) companies with asset size above the median were assigned to the LAS group and those with asset size below or equal to the median were put into SAS group. For financial leverage, companies were grouped into high financial leverage (HFL) and low financial leverage (LFL), employing a similar procedure as employed for the asset size grouping. And for operating leverage, the companies were grouped into high operating leverage (HOL) and low operating leverage (LOL), employing the same grouping procedure as above.

The reason for this stratification for selecting the sample is to investigate the effect of size, financial leverage, and operating leverage on expected rate of return of a company's stock in response to the specified hypotheses stipulated. Therefore, the sampling frame was partitioned into 3x2 = 6 cells with each cell containing companies in the same asset size subgroup, in the same financial leverage subgroup, and in the same operating leverage subgroup. From each cell in the sampling frame 200 companies were randomly selected and thus six subsamples, the SAS, the LAS, the HFL, the LFL, the HOL, and the LOL subsamples were formed. Because the companies in each cell have similar characteristics with respect to asset size, financial leverage, and operating leverage, simple random sampling method was utilized to select a sample from each cell. The number 200 for the sample size for each cell was chosen on the basis of the R-square value obtained in the pilot study conducted by this author, which will be described in this section.

The six subsamples, each containing 200 companies, were used for testing hypotheses two, three, and four. Finally, the six subsamples were merged together to come up with the total sample. Considering that some companies appeared in more than one subsample, and after removing multiple counts, the total sample size came to be 855 companies. The total sample was used for testing of hypothesis one and hypothesis five.

Sample Period

Time-series data on the selected companies were collected from January 1995 to December 2004. This time period is chosen because (a) as was discussed in the literature review chapter most tests of hypotheses already done on CAPM cover periods prior to the 1995 and their results are more or less in agreement (b) not many tests have been done for post-1995, and in particular there seems to be no empirical study on CAPM distinctly covering the second half of the 1990s and post-2000 where general economic situations were different form pre-1995 and was labeled as the era of *the new economy* and information communication revolution ,and (c) in this study it is intended to investigate validity of the standard CAPM and the multifactor model developed for the whole time span of the research and compare the results.

The Pilot Study

The purpose of the pilot study is to determine the appropriate sample size for the correlation and regression test of hypotheses, that is, for testing hypotheses one and five. For the purpose of this pilot study two companies were selected from different industries. Companies selected share similarities in certain aspects and differences in some other features. The idea is to apply the standard CAPM to these companies and to see, despite their similarities and differences, to what extent the expected rates of return of the stock of these companies are explained by the independent variable, the market rate of return, and use the information to decide on the appropriate sample size. The companies selected were Bank of America whose stock trades under the symbol *BAC* and General Motors with stock symbol *GM*. The selected companies are similar in the sense that (a) they are both included in the S&P500 Stock Index, (b) being in the S&P500 they are both regarded as large companies with high market capitalization values, and (c) they both pay out dividends. The differences between the selected companies are (a) they have different dividend pay-out ratios, (b) they have different price to earnings (P/E) ratios, (c) they come from different industries with different operating leverages, and (d) they have different financial leverages.

The single-index version of the standard CAPM was employed using S&P 500 Index as proxy for the general stock market. Monthly rates of return were calculated both for the two stocks and for the S&P 500 Index from January 2000 to December 2004 and the data were used to estimate the regression coefficients of the standard CAPM using Equation 40 below:

$$R_{jt} - R_{ft} = \alpha_j + \beta_j (R_{tM} - R_{ft}) + e_{jt} \qquad (40)$$

The results are reported in Tables 4 and 5. As Table 4 below shows the correlation between GM monthly risk premiums and S&P500 monthly risk premiums is statistically significant because the value of *F* significance in the *ANOVA* result is zero, though the *R-square* is only 31% implying that only 31% of variations in the GM monthly risk premiums is due to S&P500 and the rest is due to some other risk factors.. As for the results of the regression line, the value of *y* intercept estimate is zero, which is consistent with Equation (40), though it is not statistically significant due to its high *p* value. However, estimate of the slope, which is the *beta* and is equal to 1.24, is statistically significant with *p* value of zero.

Table 4

Regression Statistics for GM risk Premium against S&P 500 Index Risk Premium

Regression Statistics	
Multiple R	0.55
R Square	0.31
Adjusted R Square	0.30
Standard Error	0.09
Observations	59

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.20	0.20	25.32	0.00
Residual	57	0.45	0.01		
Total	58	0.65			

Regression Line	*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*
Intercept	0.00	0.01	-0.07	0.94	-0.02	0.02
S&P Risk Premium	1.24	0.25	5.03	0.00	0.75	1.74

As Table 5 below shows the correlation between BAC monthly risk premiums and S&P500 monthly risk premiums is statistically significant at above 1% because the value of F-significance in the ANOVA result is 0.06%, though the R-square is very small at 18.8% implying that only 18.8% of variations in the BAC monthly risk premiums is due to S&P500 and the rest is due to some other risk factors. As for the results of the regression line, the value of y-intercept estimate is close to zero, which is consistent with Equation 40, and with the *p* value of 1.92% is statistically significant at 2% level. Also for the estimate of the slope, which is the beta and is equal to 0.67, the *p* value is only 0.06% meaning that the slope estimate is statistically significant at 1% level.

Table 5

Regression Statistics for BAC Risk Premium against S&P500 Index Risk Premium

Regression Statistics	
Multiple R	0.43
R Square	0.19
Adjusted R Square	0.17
Standard Error	0.66
Observations	59

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.06	0.06	13.21418	0.0006
Residual	57	0.25	0.00		
Total	58	0.30			

Regression Line	*Coefficients*	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*
Intercept	0.02	0.01	2.41	0.0192	0.00
S&P Risk Premium	0.67	0.18	3.64	0.0006	0.30

The outcome of the pilot study indicates that although the empirical application of the standard CAPM gives statistically significant beta coefficients, but the explanatory power of the market portfolio as the only independent variable is not very strong, which warrants inclusion of other possible independent variables. To use this information to determine the sample size, the concept of *statistical power of the test* is utilized here. The power of the test is defined as the probability of rejecting the null hypothesis when it is false. For regression analysis, the power of the test is related to the coefficient of correlation and the sample size and the relationship is tabulated in statistical tables for various values of the related variables. The low end of the R-square from the pilot study was 18% which corresponds to a correlation coefficient of around 0.42. To achieve 90% power of the test for this value of the correlation coefficient, statistical table shows sample size should be 60. For the time-series regression on individual stocks in this study the time period is from January 1995 to December 2004 with the two subperiods of 1995-2000, and 2000-2004. Therefore, for each subperiod there are five years of monthly data which result in 60 monthly rates of return for every stock in the sample, meeting the 90% power of the test. As for the crosssection regression test and the z tests for two population means, the sample size of 200 companies chosen for this study far exceeds the sample size advised by the statistical table.

Data Collection

Data collection process does not require use of questionnaires or interviews as the study uses existing data. Required data were obtained from various sources. Monthly stock prices and quarterly dividend payments for every company in the sample and for every month of the sample period were retrieved from Yahoo -finance website (http://finance.yahoo.com). Companies' balance sheets were retrieved from Securities and Exchange Commission (SEC) website (http://sec.gov). Finally, time series data on the stock market volatility index (IV_t) were retrieved from Chicago Board of Options Exchange (CBOE) website (http://www.cboecom). Collected data were recorded in Excel worksheets and the various features of Excel were utilized to analyze the data and test the hypotheses.

Data Analysis

Data analysis involves test of hypotheses for group differences, the existence and form of relationships between variables, and verification of statistical significance of correlation and regression coefficients. Sample statistics required for data analysis are calculated from the collected data and reported in tables, histograms, and charts. In chapter 4 the results of data analysis are reported in tables containing estimated correlation and regression coefficients, relevant test statistics, and the significance levels (*p-values*). In chapter 5, results are interpreted and the standard CAPM and the multifactor model under study are assessed on the basis of the findings.

The Variables

The variables in this study and their operational definitions are as the following:

R_{jt} : Realized rate of return of the stock of company *j* during the month ***t*.** This is measured by the formula: $R_{jt} = \frac{(P_{jt} + D_{jt}) - P_{j(t-1)}}{P_{j(t-1)}}$, where P_{jt} and $P_{j(t-1)}$ are company *j*'s stock closing prices at the last trading day of the months *t* and *t-1* respectively and D_{jt} is the dividend per share paid by company *j* to its common stockholders during the month *t*. The return interval, or holding period, is chosen to be one month, rather than shorter (one day) or longer (one year), to allow enough time for the independent variables of the model to change and at the same time obtain more return observations to have smaller sampling error for the regression coefficients.

R_{ft} : Risk-free rate during the month *t***.** This is the rate of return earned on a risk-free asset. In this study the rate of return on U.S. government 3-months Treasury bills during the month ***t*** is taken as the measure of risk-free rate for the month.

R_{Mt} : Market rate of return during the month *t***.** This is the weighted average rate of return on all the stocks trading in the stock market. In this study the S&P 500 Stock Index, which is a value-weighted stock index, is chosen as the proxy for the stock market. Therefore, R_{Mt} is calculated using the same formula as used for calculating the rate of return of individual stocks except that instead of stock price the index value is put into the formula and the dividend item would be the sum of dividends paid by all the companies comprising the index.

IV_t : The stock market implied volatility for the month *t:* Implied volatility of a stock or stock index is a number derived from Black-Scholes option pricing model and is a measure of investors' attitude towards future risk of stock or the stock market. Stock options are legal contacts on an individual stock or on a stock index. A call option on a stock or on an index gives the buyer the right to purchase the stock or the index at a specified price within a specified time period and create the obligation for the seller to deliver if the buyer exercises its right. The buyer of a call option pays a premium for the call (call price) and instead receives the right to purchase a stock or index at a specified price (exercise price) within a specific time period (time to maturity). Black and Scholes

(1973, pp.637-659) developed a sophisticated mathematical model in which they derived the price of a call option as the function of exercise price, time to maturity, risk-free rate, and the standard deviation (volatility) of the stock's or stock index's rate of return. If one takes the market price of the call option as a given datum, then working out Black-Scholes formula backward to find the standard deviation (volatility) gives what is known as implied volatility. As option contacts are forward looking instruments in the capital market, one can regard the implied volatility of a general stock index as the overall investors' expectation for the future risk of the stock market. Therefore, in the model developed in this study it is hypothesized that the higher implied volatility at time *t-1*, the lower would be general stock market prices at time *t-1* and thus the higher the rate of return of stocks at time *t*. In other words, implied volatility is a systematic factor positively correlated with stock returns with one period time lag. Currently the implied volatility of the US stock market is measured and reported by the Chicago Board of Option Exchange (CBOE) on two stock indices. The VIX as a measure of the stock market volatility is calculated based on call options on S&P 100 Stock Index, and the VNX is calculated based on call options on NASDAQ 100 Stock Index. In this study the VIX is used for the stock market's implied volatility, IV_t, because S&P 100 is a more diversified index than the NASDAQ100, which makes the VIX a better representative of the general stock market's risk expectation than the VNX.

$\overline{R}_k^{LAS}$: Average monthly rate of return of the large company *k* during the whole sample period (1995-2004). This is calculated by finding the simple monthly average rate of return of every company in the LAS subgroup for the sample period.

$\overline{R}_j^{SAS}$: Average monthly rate of return of the small company *j* during the whole sample period. This is calculated by finding the simple monthly average rate of return of every company in the SAS subgroup for the sample period.

SLL_t : The difference between the average rate of return of large companies and the average rate of return of small companies during the month *t*. This is calculated by finding the simple average rate of returns of all companies in the LAS subgroup during period *t* and subtracting it from simple average rate of returns of all companies in the SAS subgroup during period *t*.

$\overline{R}_k^{HFL}$: Average monthly rate of return of the high financial leverage company *k* during the sample period (1995-2004). This is calculated by finding the simple monthly average rate of return of every company in the HFL subgroup for the sample period. Financial leverage is defined as the company's degree of indebtedness, measured by dividing total long-term debts (debts over one year maturity) of the company by its total assets.

$\overline{R}_j^{LFL}$: Average monthly rate of return of the low financial leverage company *j* during the sample period. This is calculated by finding the simple monthly average rate of return of every company in the LFL subgroup for the sample period.

$HFLLF_t$: The difference between the average rate of return of high financial leverage companies and the average rate of return of low financial leverage companies during the month ***t*.** This is calculated by finding the simple average rate of returns of all companies in the HFL subgroup during period ***t*** and subtracting it from simple average rate of returns of all companies in the LFL subgroup during period *t*.

$\overline{R}_k^{HOL}$: Average monthly rate of return of the high operating leverage company k during the sample period (1995-2004). This is calculated by finding the simple monthly average rate of return of every company in the HOL subgroup for the sample period. Operating leverage is the company's level of fixed costs in relation to total costs of operations and in this study is measured by dividing net fixed assets by total assets.

$\overline{R}_j^{LOL}$: Average monthly rate of return of the low operating leverage company j during the sample period (1995-2004). This is calculated by finding the simple monthly average rate of return of every company in the LOL subgroup for the sample period.

$HOLLO_t$: The difference between the average monthly rate of return of high operating leverage companies and the average rate of return of low operating leverage companies during the month t. This is calculated by finding the simple average rate of returns of all companies in the HOL subgroup during period t and subtracting it from simple average rate of returns of all companies in the LOL subgroup during period t.

The Hypotheses

The hypotheses that are tested in this study pertain to the relationships between rates of return on stocks of public companies and relevant risk factors. Two types of relationships are examined and test of hypotheses are performed to verify statistical significance of the relationships. First, the sample companies are separately grouped into high (H) and low (L) subgroups based on size, financial leverage, and operating leverage and the subgroups are compared to test if there are any significant *differences among groups* on the dependent variable. Second, the numerical relationship between the independent variables and the dependent variable and its statistical significance is studied through multivariate *correlation* and *regression* analysis. This leads to the five hypotheses as stated below. Hypothesis one is to test the validity of standard CAPM for the sample period, January 1, 1995 to Dec 31, 2004. Hypotheses two, three, and four are this author's propositions to bridge the field of investment theory to the field of corporate finance theory. And hypothesis five is this author's proposed contribution to the field.

Hypothesis one: The market factor is the only factor that explains variation of returns across different stocks and the more sensitive a stock rate of return with respect to the market factor, the higher the rate of return on the stock. This is to test the standard single-factor CAPM and it involves testing the following regression models for significance of correlation and regression coefficients:

$$R_{jt} - R_{ft} = \alpha_j + \beta_j (R_{tM} - R_{ft}) + e_{jt}$$

$$\overline{R_j - R_f} = \lambda_0 + \lambda_1 b_j + \lambda_2 \sigma^2 (e_j) + e'_j$$

Using the language of test of hypothesis, the null and alternate hypotheses for the first equation of hypothesis one would be:

$$H_0 : \alpha_j, \beta_j = 0$$
$$H_1 : \alpha_j, \beta_j \neq 0$$

, and for the second equation of hypothesis one the null and the alternate would be:

$$H_0 : \lambda_0 = 0, \lambda_1 = \overline{R_M - R_f}, \lambda_2 = 0$$
$$H_1 : \lambda_0 \neq 0, \lambda_1 \neq \overline{R_M - R_f}, \lambda_2 \neq 0$$

The first equation reflects the first CAPM proposition that the market factor is the risk factor in explaining rate of returns of stocks and estimates the responsiveness of every stock *j* during the sample period. If the standard CAPM is true then the intercept a_j, which is the regression estimate for α_j, should not be significantly different from zero; the slope b_j, which is the regression estimate for β_j, should be significantly different from zero; and the correlation should yield a high R-square value. The second equation reflects the CAPM proposition that expected rates of return on stocks are directly related to their systematic risks, betas, and that the nonsystematic risk is not important. If this is true then the second regression equation should yield $\lambda_0 = 0$, $\lambda_1 = \overline{R_M - R_f}$, and $\lambda_2 = 0$.

Hypothesis two: Expected (average) monthly rate of return of small stocks is higher than expected (average) monthly rate of return of large stocks. This is based on the corporate finance proposition that investors regard small companies to be more risky than large companies as small companies face more *business risk* than the large companies. The following *z* test for two population means is conducted to test hypothesis two:

$$H_0 : \mu(\overline{R}_j^{SAS}) \leq \mu(\overline{R}_k^{LAS})$$
$$H_1 : \mu(\overline{R}_j^{SAS}) > \mu(\overline{R}_k^{LAS})$$

Hypothesis three: Expected (average) monthly rate of return of high financial leverage stocks is higher than expected (average) monthly rate of return of low financial leverage stocks. This is based on the corporate finance proposition that investors regard high financial leverage companies to be more risky than low financial leverage companies as high financial leverage companies face more *financial risk* than the low financial leverage companies. The following *z* test for two population means is conducted to test hypothesis three:

$$H_0 : \mu(\overline{R}_j^{HFL}) \leq \mu(\overline{R}_k^{LFL})$$
$$H_1 : \mu(\overline{R}_j^{HFL}) > \mu(\overline{R}_k^{LFL})$$

Hypothesis four: Expected (average) monthly rate of return of high operating leverage stocks is higher than expected (average) monthly rate of return of low operating leverage stocks. This is based on the corporate finance proposition that investors regard high operating leverage companies to be more risky than low operating leverage companies as high operating leverage companies face more *business risk* than the low operating leverage companies. The following *z* test for two population means is conducted to test hypothesis four:

$$H_0 : \mu(\overline{R}_j^{HOL}) \leq \mu(\overline{R}_k^{LOL})$$
$$H_1 : \mu(\overline{R}_j^{HOL}) > \mu(\overline{R}_k^{LOL})$$

Hypothesis five: This hypothesis is this author's proposed contribution to the field. It is a multifactor model and posits that (a) the expected rate of return of any stock is linearly dependent on five risk factors, the market return, the market implied volatility, the company's asset size, the company's financial leverage, and the company's operating leverage and (b) expected rates of return across cross section of stocks are linearly related to coefficients of risk factors estimated in part (a). Part (a) of hypothesis five is tested through:

$$R_{jt} - R_{ft} = \alpha_j + \beta_j^M (R_{tM} - R_{ft}) + \beta_j^{IV}(IV_{t-1}) + \beta_j^S (SLL_t) + \beta_j^{FL}(HFLLF_t) + \beta_j^{OL}(HOLLO_t) + e_{jt}$$

where, the β_j's are the responsiveness of stock *j* rate of return to the specified risk factors. Using the language of test of hypothesis, the null and alternate hypotheses for the first equation of hypothesis five would be:

$$H_0 : \alpha_j, \beta_j^M, \beta_j^{IV}, \beta_j^S, \beta_j^{FL} = 0$$
$$H_1 : \alpha_j, \beta_j^M, \beta_j^{IV}, \beta_j^S, \beta_j^{FL} \neq 0$$

Part (b) of hypothesis five is tested through:

$$\overline{R_j - R_f} = \lambda_0 + \lambda_1 b_j^M + \lambda_2 b_j^{IV} + \lambda_3 b_j^S + \lambda_4 b_j^{FL} + \lambda_5 b_j^{OL} + e_j$$

And the null and alternate hypotheses:

$$H_0 : \lambda_0 = 0,\ \lambda_1 = \overline{R_M - R_f},\ \lambda_2 = \overline{IV},\ \lambda_3 = \overline{SLL},\ \lambda_4 = \overline{HFLLF},\ \lambda_5 = \overline{HOLLO}$$

$$H_1 : \lambda_0 \neq 0,\ \lambda_1 \neq \overline{R_M - R_f},\ \lambda_2 \neq \overline{IV},\ \lambda_3 \neq \overline{SLL},\ \lambda_4 \neq \overline{HFLLF},\ \lambda_5 \neq \overline{HOLLO}$$

where, the b_j's are estimates of the β_j's found from the first regression equation.

Summary

This research is about the existence and nature of the relationship between the rates of return on stocks and the risk factors for which investors in the stocks want to be compensated for. Markowitz portfolio selection theory was the first attempt to study the relationship between the rates of return and risks of securities. Based on the assumption that investors are risk averse and prefer less risk to more risk for the same expected return, Markowitz's model provides guidelines for the investors as to how to construct optimal portfolios given their knowledge of expected return and risk of every security in the market and utilize the benefits of diversification. However, the model does not explain how investors form their return expectations about individual securities in the first place. Therefore, Markowitz model can not serve as a theory of equilibrium prices in the capital market.

The CAPM extends Markowitz model by introducing the risk-free asset which investors can combine it with a portfolio on the efficient frontier through lending and borrowing at the fixed risk-free rate. With this addition and by positing that rational investors would select portfolios that yield highest expected return in excess of the risk-free rate per unit of risk taken, CAPM concludes that there exists one specific portfolio on the efficient frontier that could serve as the optimal portfolio for all investors. That single optimal portfolio consists of all risky assets weighted by their market capitalization and is labeled the market portfolio. From here, the famous expected return-beta relationship of CAPM is deduced according to which the expected excess return of any security is linearly related to the beta of the security and the expected excess return on the market portfolio. The expected return-beta relationship is the basis of equilibrium asset prices in the capital market and if any deviation from equilibrium prices occur in the market the mean variance efficient investors acting on the basis of expected return-beta relationship will readjust their portfolios and restore equilibrium prices.

Empirical tests of the standard CAPM using realized or *ex post* returns as proxies for the expected or *ex ante* returns do not lead to consensually unambiguous conclusions with respect to the expected return-beta relationship as posited by the standard CAPM. Furthermore, these tests do not find the relationship between the beta, as the only risk factor in the standard CAPM, and the realized rate of return to be statistically very strong. But the model has been employed as a useful analytical tool and has survived as a practical aid to financial decision maker over the past 30 years. It is therefore worthwhile to extend the model along some lines more consistent with empirical observations while preserving the basic ideas of the model, especially the intuitively appealing view that return and risk are related.

In this dissertation the author suggests to develop and test a model that addresses both of the above-mentioned shortcomings of the standard CAPM. First, unlike the standard CAPM that is a single-factor model, the proposed model is multifactor incorporating other explanatory variable that reflect investors' expectations as well as company-specific risk factors. The implied volatility of the stock market, computed from the Black-Scholes option-pricing model and measured and reported by the Chicago Board of Option Exchange (CBOE), is proposed as an additional systematic risk factor which accounts for the *ex ante* nature of the return expectations. To reconcile the concept of risk factors as it comes in the field of corporate finance theory with that of the field of investment theory, three company-specific risk factors are included in the proposed model. These risk factors are the company's asset size, financial leverage, and operating leverage. It is expected that this multifactor model will provide a better explanation of the variation of realized returns across different stocks than the standard single-factor CAPM.

CHAPTER 4:

RESULTS AND FINDINGS

Introduction

This chapter's focus is on testing of the five hypotheses proposed in chapter 3. Each hypothesis is separately restated and the appropriate statistical method is applied for testing validity of the hypothesis. Under each hypothesis, the type of data used for testing of that hypothesis is described and the findings of the test are reported. The final section of this chapter contains summary of the results and findings.

Data Analysis

In this section, each of the five hypotheses proposed in chapter 3 are restated. Under each hypothesis, the required data for testing of that hypothesis are specified, appropriate descriptive statistics are calculated and presented in tables, relevant type of hypothesis testing are conducted, and the results of the test are reported.

Hypothesis One

Hypothesis one is an expression of the standard single-factor CAPM. It posits that the market factor is the only factor that explains variation of returns across different stocks and the more sensitive a stock rate of return with respect to the market factor, the higher the rate of return of the stock. Hypothesis one involves testing the following two regression models for significance of correlation and regression coefficients:

$$R_{jt} - R_{ft} = \alpha_j + \beta_j (R_{Mt} - R_{ft}) + e_{jt} \qquad (41)$$

$$\overline{R_j - R_f} = \lambda_0 + \lambda_1 b_j + \lambda_2 \sigma^2(e_j) + e'_j \qquad (42)$$

where, R_{jt} is the realized rate of return on stock *j* during the month *t*, R_{ft} is the risk-free rate during the month *t*, and R_{Mt} is the rate of return on the market portfolio during the month t , $\overline{R_j - R_f}$ is average monthly risk premium of stock *j* during the study period (average monthly rate of return of stock *j* less average monthly risk-free rate during the study period of 120 months), e_{jt} is the error term for stock *j*'s rate of return in month *t*, and $\sigma^2(e_j)$ is the variance of stock *j*'s error term during the study period. In the language of regression analysis the term $\sigma\ (e_j)$ is called *standard error* of the regression and in Equation 41 it represents the amount of nonsystematic risk of stock *j*'s rate of return.

Using the language of test of hypothesis, the null and alternate hypotheses for the first equation of hypothesis one would be:

$$H_0 : \alpha_j, \beta_j = 0$$
$$H_1 : \alpha_j, \beta_j \neq 0$$

, and for the second equation of hypothesis one the null and the alternate would be:

$$H_0 : \lambda_0 = 0, \lambda_1 = \overline{R_M - R_f}, \lambda_2 = 0$$
$$H_1 : \lambda_0 \neq 0, \lambda_1 \neq \overline{R_M - R_f}, \lambda_2 \neq 0$$

The first equation of hypothesis one reflects the first CAPM proposition that the market factor is the only risk factor in explaining time-series variations in the rates of return of every stock and estimates the responsiveness of every stock *j* to the market factor during the sample period. If the standard CAPM is true then the intercept a_j , which is the regression estimate for α_j , should not be significantly different from zero;

the slope b_j, which is the regression estimate for β_j, should be significantly different from zero; and the correlation should yield a high R-square value. The second equation reflects the CAPM proposition that at any point in time expected rates of return on cross section of stocks are directly related to their systematic risk coefficients, the betas, and that the nonsystematic risk is not important. If this is true then the second regression equation should yield $\lambda_0 = 0$, $\lambda_1 = \overline{R_M - R_f}$ (average monthly market risk premium, that is, average rate of return of the market portfolio less average risk free -rate during the study period), and $\lambda_2 = 0$.

Testing of Hypothesis One: First Part

To test hypothesis one, through regression Equation 41, monthly closing prices of the stocks of all the 855 companies in the sample, and dividend payments for those companies that paid dividend, for the period January 1995 to December 31, 2004 were retrieved from Yahoo-finance database. Therefore, for each company's stock 120 monthly closing prices were collected. Price and dividend data were then used to measure the realized rate of returns of every stock and for every month of the study period using the following formula:

$$R_{jt} = \frac{(P_{jt} + D_{jt}) - P_{j(t-1)}}{P_{j(t-1)}} \qquad (43)$$

where, P_{jt} and $P_{j(t-1)}$ are company j 's stock closing prices at the last trading day of the months t and $t\text{-}1$ respectively and D_{jt} is the dividend per share paid by company j to its common stockholders during the month t. Therefore for each of the 855 stocks is the sample 119 monthly rates of return were calculated. As in this study the proxy for the market portfolio is taken to be the S&P 500 Stock Index, monthly data on S&P 500 Stock Index closing values, and dividends paid by the companies in the index were collected for the study period and Equation 43 was applied to calculate 119 monthly rates of return of the market, that is, 119 R_{Mt} 's. Finally, 3-month Treasury bill rates were collected for the 120 months of the study period. However, as the Treasury rates are reported by the Federal Reserve on an annualized basis, the rates were divided by 12 to obtain 119 monthly risk –free rates, R_{ft} 's.

To estimate the regression coefficients of Equation 41 and test for statistical significance of the estimated coefficients, related data for each company in the sample were organized in separate Excel worksheets. An example of this task for the case of Merck Company (ticker symbol MRK) is partially illustrated in Table 6 below:

Table 6

Time Series Data on Monthly Excess Returns of Merck Company's Stock and Monthly Excess Returns S&P500 Stock Index, January 1995 to December 2004

Date	Monthly returns on 3month Tbills R_{ft}	S&P 500 monthly returns R_{Mt}	MRK monthly returns R_{jt}	S&P 500 monthly excess returns $R_{Mt} - R_{ft}$	MRK monthly excess returns $R_{jt} - R_{ft}$
1-Dec-04	0.185%	3.246%	16.30%	3.061%	16.117%
1-Nov-04	0.176%	3.859%	-10.51%	3.684%	-10.686%
1-Oct-04	0.149%	1.401%	-5.13%	1.252%	-5.277%
1-Sep-04	0.140%	0.936%	-26.00%	0.796%	-26.141%
2-Aug-04	0.125%	0.229%	-0.83%	0.104%	-0.956%
1-Jul-04	0.113%	-3.429%	-4.52%	-3.542%	-4.632%
2-Jan-04	0.075%	1.728%	3.04%	1.653%	2.963%
1-Dec-03	0.076%	5.077%	14.80%	5.001%	14.719%
3-Nov-03	0.079%	0.713%	-8.26%	0.634%	-8.344%
1-Jul-03	0.077%	1.622%	-8.71%	1.546%	-8.785%
2-Jun-03	0.078%	1.132%	9.65%	1.054%	9.574%
1-May-03	0.091%	5.090%	-4.46%	4.999%	-4.554%
1-Feb-96	0.403%	0.693%	-5.54%	0.291%	-5.943%
3-Jan-96	0.417%	3.262%	6.89%	2.845%	6.473%
1-Dec-95	0.428%	1.744%	6.62%	1.316%	6.187%
1-Nov-95	0.447%	4.105%	7.58%	3.658%	7.132%
3-Apr-95	0.471%	2.796%	0.57%	2.325%	0.098%
1-Mar-95	0.478%	2.733%	1.28%	2.255%	0.802%
1-Feb-95	0.481%	3.607%	5.32%	3.127%	4.843%

Using the regression feature of Excel, the data organized in Table 6 were then used to solve the regression Equation 41 and to find the regression coefficients and their statistical significance for every stock in the sample. Regression results for the case of Merck Company (MRK) are exhibited in Table 7 below:

Table 7

Regression Output: MRK Monthly Excess Returns versus S&P 500 Monthly Excess Returns

Regression Statistics	
Multiple R	33.38%
R Square	11.14%
Adjusted R Square	10.38%
Standard Error:	8.14%
Observations	119

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	0.097	0.097	14.667	0.02%
Residual	117	0.776	0.007		
Total	118	0.873			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept:	0.003	0.75%	0.415	67.86%
S&P monthly excess returns:	0.636	16.61%	3.830	0.02%

As the regression results in Table 7 show, for the case of MRK stock the regression intercept value is .003 with *p* value of 75% which does not reject the null hypothesis that α_j for MRK is statistically equal to zero. The slope value is 0.636 with *p* value of 0.02% which leads to rejection of the null hypothesis that β_j is statistically equal to zero. The R-square value for MRK is 11.14% implying that only 11.14% of variations in the monthly rates of return of MRK are due to variations in monthly rates of return of the S&P500

Stock Index (the market) and thus about 89% of variations in MRK monthly returns are due to factors other than the market returns.

To test the first part of hypothesis one as exemplified in Equation 41, tables similar to Table 6 and Table 7 were constructed for all the 855 stocks in the sample and for each of them regression results were recorded in tables similar to Table 7. The results were then summarized on the basis of frequency distribution of the α_j and β_j statistical significance and for different ranges of adjusted R-square values. The results for α_j and β_j statistical significances are reported in Table 8 below:

Table 8

Summary of Regression Results for the 855 Stocks in the Sample: Statistical Significance of Regression Coefficients

α_j Summary of regression coefficients	At 1% level	At 5% level	At 10% level
Percentage of β_j 's significantly different from zero	56%	70%	77%
Percentage of 's significantly not different from zero	99%	84%	76%

Summary results for percentage distribution of adjusted R-square values for different ranges of adjusted R-squares are shown in Table 9 below:

Table 9

Summary of Regression Results for 855 Stocks: Percentage Distribution of R-square Values

Summary of adjuste R-square results	
Adjusted R-sqaure range	Percentage of stocks in the range
0-10%	61%
10%-20%	28%
20%-30%	5%
30%-40%	3%
40%-50%	2%
50%-60%	1%
Over 60%	0%
Average adjusted R-square	9.78%
Median adjusted R-square	6.60%

Testing of Hypothesis One: Second Part

The second part of hypothesis one, as expressed in regression Equation 42 and the related null and alternate hypotheses regards the average monthly excess returns of the stocks in the sample, the $\overline{R_j - R_f}$'s, as the dependent variable with the estimates of the betas, the b_j's, and the estimates of nonsystematic risks, the $\sigma^2(e_j)$'s, obtained form the first part, as the independent variables. Estimates of the terms $\sigma^2(e_j)$ can be obtained using the formula $\sigma_j^2 = b_j^2\sigma_M^2 + \sigma^2(e_j)$ as discussed in the literature review chapter, or it can be obtained from the regression outputs produced through Excel program. In the regression output in Microsoft Excel the square of the term *standard error* in the *regression statistics* table or the tem *MS Residual* in the *ANOVA* table both measure the variance of the regression residual term and can be taken as the estimate of stock's nonsystematic risk, $\sigma^2(e_j)$. To test the second part of hypothesis one, the average monthly rate of returns for every stock in the sample, the $\overline{R_j}$'s and average monthly risk-free rates, the $\overline{R_f}$, over the sample period (120 months) were calculated and together with the estimated b_j 's and $\sigma^2(e_j)$'s were organized in an Excel worksheet a portion of which is illustrated in Table 10 below:

Table 10

Stocks' Average Monthly Returns versus their Betas and Nonsystematic Risks (partial)

COMPANY	**SYMBOL**	$\overline{R_j}$ (%)	$\overline{R_f}$ (%)	$\overline{R_j} - \overline{R_f}$ (%)	b_j	$\sigma^2(e_j)$ (%)
ACE CASH EXPRESS INC	AACE	2.34	0.32	2.02	0.659	1.17
ARCH COAL INC	ACI	1.4	0.32	1.08	0.17	2.07
APPLIED MATERIALS INC	AMAT	2.93	0.32	2.61	2.363	1.51
AMCORE FINANCIAL INC	AMFI	1.22	0.32	0.9	0.233	0.40
AUTONATION INC	AN	4.3	0.32	3.98	1.199	8.88
ANNTAYLOR STORES CORP	ANN	1.62	0.32	1.3	1.21	2.30
ASTA FDG INC	ASFI	6.05	0.32	5.73	0.659	10.43
APTAR GROUP INC	ATR	1.61	0.32	1.29	0.436	0.60
AMERICAN EXPRESS CO	AXP	1.89	0.32	1.57	1.241	0.29
BURLINGTON COAT FACTORY	BCF	1.69	0.32	1.37	0.626	1.45
BENCHMARK ELECTRONICS	BHE	2.36	0.32	2.04	1.21	2.08
BJS RESTAURANTS INC	BJRI	2.74	0.32	2.42	0.564	2.39
BANKUNITED FINANCIAL	BKUNA	2.01	0.32	1.69	0.312	1.11
BELLSOUTH CORP	BLS	1.1	0.32	0.78	0.831	0.57
BURLINGTON NORTHERN	BNI	1.38	0.32	1.06	0.65	0.41
BARR PHARMACEUTICAL INC	BRL	3.19	0.32	2.87	0.749	1.77
BASSETT FURNITURE IND	BSET	0.63	0.32	0.31	0.686	0.85
CEHALON INC	CEPH	3.72	0.32	3.4	1.148	4.79
O CHARLEYS INC	CHUX	1.34	0.32	1.02	0.593	1.01

The data in Table 10 were then used to solve the regression Equation 42 and thus test the second part of hypothesis one. As was pointed out before, part two of hypothesis one posits that (a) expected or average monthly rate of returns of stocks in excess of the expected (average) monthly risk-free are linearly and positively dependent on their betas,

(b) the intercept of the line relating the two passes from the origin, and (c) beta, or the measure of systematic risk , is the only factor that causes variations in the expected monthly rates of return of a cross section of stocks and the nonsystematic risk does not affect expected returns of the stocks because it is diversifiable through portfolios. The results of the regression conducted through Microsoft Excel appear in Table 11 below:

Table 11

Regression Output: Stocks' Average (Expected) Monthly Returns versus Systematic and Nonsystematic Risks

Regression Statistics	
Multiple R	0.802
R Square	64.40%
Adjusted R Squar	64.33%
Standard Error	0.84%
Observations	855

ANOVA

	df	*SS(%)*	*MS (%)*	*F*	*P-value*
Regression	2	10.14	5.07	724.28	0.00%
Residual	852	5.60	0.007		
Total	854	15.74			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	0.85%	0.13%	6.75	0.00%
β_j	0.59%	0.13%	4.41	0.00%
$\sigma^2(e_j)$	0.26	0.03	9.93	0.00%

According to the regression output presented in Table 11, the coefficient of multiple regression for Equation 42 is 0.80 with *p* value of 0.00% implying that the dependent variable is strongly correlated with combination of the two independent variables.

To determine if the estimates of the regression coefficients are close to their hypothesized values it is noted that during the study period (120 months) the average monthly risk-free rate ($\overline{R_f}$) was 0 .32% and the average monthly return on S&P 500 stock Index ($\overline{R_M}$) was 0.90%, that is, average monthly excess return of the market was 0.58%.. This information and the information provided in Table 11 are combined in Table 12 below to be utilized for testing of the second part of hypothesis one.

Table 12

Regression Results for Test of Hypothesis One: Second Part.

	λ_0	λ_1	λ_2
Coefficient	0.85%	0.59%	0.26
Hypothesized Value	0.00%	0.58%	0.00
Standard Error	0.13%	0.13%	0.03
t Statistic	6.55	0.08	8.67
P-value	0.00%	93.63%	0.00%

Sample average for annual excess market return, $\overline{\boldsymbol{R}_M - \boldsymbol{R}_f} = 0.58\ \%$
Adjustedd R-square, 64.33%

As per the information in Table 12, regression estimate for λ_0 is 0.85% with standard error of 0.13%. This means that the estimate of λ_0 is away from its hypothesized value of zero by 6.55 standard error (t = 6.55, p value = 0). Therefore, the null hypothesis that $\lambda_0 = 0$, is rejected. The regression estimate for λ_1 is 0.59% with standard error of 0.13% which makes it to be different from its hypothesized value of 0.58% by only 0.08 standard error of the estimate (t = 0.08, p value = 93.63%). Thus, the null hypothesis that $\lambda_1 = \overline{R_M - R_f}$ =0.58% is not rejected. Finally, the regression estimate for

λ_2 is 0.26 with standard error of 0.03. This makes the estimate of λ_2 to be different from its hypothesized value of zero by 8.67 standard error (t = 8.67, *p* value = 0).Therefore, the null hypothesis that $\lambda_2 = 0$ is rejected.

The finding that λ_1 is not significantly different from 0.58% (the market average monthly risk premium for the study period) means that λ_1 is statistically different from zero and this supports the CAPM proposition that systematic risk, the beta, is a significant factor in determining expected rates of return of stocks. However, the finding that λ_2 is also significantly different from zero indicates that nonsystematic risk is also a significant factor in determining the stocks' expected returns. Moreover, the adjusted R-square value of 64% points to the existence of other risk factors, systematic or nonsystematic, that affect the expected rates of return of the stocks. The purpose in hypothesis two through hypothesis five is to look for some of such possible risk factors.

Hypothesis Two

The purpose in hypothesis two is to verify the effect of a company's size, as measured by market value of its assets, on the rate of return of the company's stock. Therefore, in hypothesis two it is intended to find out if the nonsystematic risk factor, *size*, has effects on the stocks' average rates of return. Here, the null and alternate hypotheses are stipulated on the basis of corporate finance proposition that investors regard small companies to be more risky than large companies; as small companies face more *business risk* than large companies. If this is true then the rates of return of stocks of small companies should be on average greater than the rates of return of stocks of large companies. This proposition can be tested through the following null and alternate hypotheses:

$$H_0 : \mu(\overline{R}_j^{SAS}) \leq \mu(\overline{R}_k^{LAS})$$
$$H_1 : \mu(\overline{R}_j^{SAS}) > \mu(\overline{R}_k^{LAS})$$

where, $\overline{R}_j^{SAS}$ is average monthly rate of return of the small company *j* over the study period (average rate of return over 120 months), $\overline{R}_k^{LAS}$ is average monthly rate of return of the large company *k* over the study period, $\mu(\overline{R}_j^{SAS})$ is the mean of all small companies average rates of return, and $\mu(\overline{R}_j^{LAS})$ is the mean of all large companies average rates of return.

Testing of Hypothesis Two

The first step taken in testing hypothesis two was to measure the asset size of companies in the sampling frame, averaged over the study period. The companies' asset sizes were measured by calculating the market value of the companies' assets at the beginning of the study period (Dec 31, 1994) and at the end of the study period (Dec 31, 2004) and then averaging the two. To measure the market value of the assets of a company at the beginning of the period the following calculations were made (a) the *market capitalization* of the company was calculated by multiplying the number of basic outstanding common shares of the company as of Dec 31, 1994 by the market price of its common stock at the same date and (b) the market capitalization was added to the total liabilities of the company as of Dec 31, 1994. The same calculations were done to measure the market value of the companies' assets for the end of the study period Dec 31, 1994. Number of basic common shares outstanding was taken from the companies' income statements at relevant dates. In some cases where number of outstanding shares was not reported in the income statement, the companies' net income was divided by income per basic common shares outstanding (EPS) to arrive at the number of basic common shares outstanding. The total liabilities items were taken from the companies' balance sheets at relevant dates. An example of the procedure for measuring a company's average asset size during the study period is illustrated in Table 13 below.

Table 13

Total Assets Market Value Measurement for Hewlett Packard Company (HPQ)

	Beginning of period	End of period	Average
Share price($)	100.5	20.97	
Number of outstnding common shares($m)	261	3037	
Market capitalization($m)	26230.5	63685.89	
Total liabilities($m)	18082	35670	
Total assets market value($m)	44312.50	99355.89	**71834.20**

To prepare for testing of hypothesis two, the following procedure were taken (a) tables similar to Table 13 were worked out for every company in the sampling frame, (b) companies in the sampling frame were ordered on the basis of average total assets market value from the largest to the smallest, (c) from the list of companies with asset size above the median asset size, 200 companies were randomly selected and were designated as the large asset size (LAS) subsample, (d) from the list of companies with asset size below the median asset size, 200 companies were randomly selected and were designated as the small asset size (SAS) subsample, (e) in each subsample, average monthly rate of return of every company's stock were calculated for the sample period (120 months) and the results were classified under each subsample. A portion of LAS versus SAS companies' monthly returns is exhibited Table 14 below.

Table 14

Average Monthly Rates of Return of Stocks of Small Companies versus Large Companies (portion)

Small companies' average monthly returns(%) $\overline{R}_j^{SAS}$	Large companies' average monthly returns(%) $\overline{R}_k^{LAS}$
4.9	1.11
2.44	1.89
3.73	2.14
3.54	1.53
4.24	2.21
2.74	2.63
3.51	5.21
3.13	2
3.08	3.44
6.05	1.86
2.26	1.29
2.29	3.72
5.8	1.25

The *z* test technique was applied to the data in Table 14 to verify if the mean monthly rate of returns of small companies' stocks is greater than the mean monthly rate of returns of large companies' stocks. The result of the *z* test is illustrated in Table 15 below. The results in Table 15 indicate that the mean for the average monthly returns of small companies over the study period was 2.64% with variance of 0.075% and the mean for the average monthly returns of large companies over the study period was 1.93% with variance of 0.77%. These combinations of mean-variance lead to the *z* statistic value of 3.5 which gives the *p* value of 0.02% for the one-tail test. Therefore, the null hypothesis is rejected and the alternate hypothesis is accepted, meaning that based on the sample results the average monthly rates of return of small companies' stocks is higher than the average monthly rates of return of large companies' stocks; with the difference being 0.71%.

Table 15

Results of the Z Test for Comparing Small and Large Companies' Average Returns

z-Test: Two Sample for Means		
	$\overline{R}_j^{SAS}$	$\overline{R}_k^{LAS}$
Mean	2.64%	1.93%
Known Variance	0.075%	0.770%
Observations	200.00	200.00
Hypothesized Mean Differenc	0.00	
z	3.50	
P(Z<=z) one-tail	0.02%	
z Critical one-tail	1.64	
P(Z<=z) two-tail	0.04%	
z Critical two-tail	1.96	

Hypothesis Three

The purpose in hypothesis three is to verify the effect of a company's financial leverage on the rates of return of the companies' stocks. A company's financial leverage is measured by the ratio of its long-term debts to the book value of its total assets. Therefore, in hypothesis three it is intended to find out if the nonsystematic risk factor, *financial leverage*, has effects on the stocks' average rate of returns. Here, the null and alternate hypotheses are stipulated on the basis of corporate finance proposition that investors regard high financial leverage companies to be more risky than low financial leverage companies because high financial leverage companies face more *financial risk* than low financial leverage companies. If this is true then the rates of return of stocks of high financial leverage companies should be on average greater than the rates of return of stocks of low financial leverage companies. This proposition can be tested through the following null and alternate hypotheses:

$$H_0 : \mu(\overline{R}_j^{HFL}) \leq \mu(\overline{R}_k^{LFL})$$
$$H_1 : \mu(\overline{R}_j^{HFL}) > \mu(\overline{R}_k^{LFL})$$

where, $\overline{R}_j^{HFL}$ is average monthly rate of return of the high financial leverage company ***j*** over the study period (average rate of return over 120 months), $\overline{R}_k^{LFL}$ is average monthly rate of return of low financial leverage company *k* over the study period, $\mu(\overline{R}_j^{HFL})$ is the mean of all high financial leverage companies average rate of returns, and $\mu(\overline{R}_j^{LFL})$ is the mean of all low financial leverage companies average rate of returns.

Testing of Hypothesis Three

The first step taken in testing hypothesis three was to measure the financial leverage of companies in the sampling frame, averaged over the study period. The companies' financial leverages were measured by calculating the ratio of the companies' long-term debt by the book value of the companies' total assets at beginning of the study period (Dec 31, 1994) and at the end of the study period (Dec 31, 2004) and then averaging the two. An example of the procedure for measuring a company's average financial leverage during the study period is illustrated in Table 16 below.

Table 16

Financial Leverage Measurement for Hewlett Packard Company (HPQ)

	Beginning of period	End of period	Average
Long-term debts($m)	572	4907	
Total assets book value($m)	18503	74381	
Financial leverage	3.09%	6.60%	**4.84%**

To prepare for testing of hypothesis three, the following procedures were taken (a) tables similar to Table 16 were worked out for every company in the sampling frame, (b) companies in the sampling frame were ordered on the basis of average financial leverage form the highest to the lowest, (c) from the list of companies with average financial leverage above the median financial leverage, 200 companies were randomly selected and were designated as the high financial leverage (HFL) subsample, (d) from the list of companies with average financial leverage below the median financial leverage, 200 companies were randomly selected and were designated as the low financial leverage (LFL) subsample, (e) in each subsample, average monthly rates of return of every company's stock were calculated for the sample period (120 months) and the results were classified under each subsample. A portion of HFL versus LFL companies' monthly returns is exhibited in Table 17 below. The z test technique was applied to the data in Table 17 to verify if the mean monthly return of stocks of low leverage companies is greater than that of high leverage companies. The result of the z test is illustrated in Table 18 below.

Table 17

Average Monthly Rates of Return of Stocks of Low Financial Leverage Companies versus those of High Financial Leverage Companies (Portion)

Average monthly returns(%) Low financial leverage companies $\overline{R}_k^{LFL}$	Average monthly returns(%) High financial leverage companies $\overline{R}_j^{HFL}$
3.51	0.85
3.73	1.26
1.98	1.69
2.35	1.89
3.39	1.42
1.38	1.69
2.52	0.86
6.21	1.78
2.89	1.61
1.36	2.48
1.52	3.52
3.54	2.05

.

The results in Table 18 indicate that the mean for the average monthly returns of low financial leverage companies over the study period was 2.29% with variance of 0.015% and the mean for the average monthly returns of high financial leverage companies over the study period was 2.28% with variance of 0.70%. These combinations of mean-variance lead to the z statistic value of 0.004 which gives the p value of 49.8% for the one-tail z test. Therefore, the null hypothesis is not rejected and the alternate hypothesis is rejected, meaning that based on the sample results the average monthly rates of return of high leverage companies' stocks is *not* higher than the average monthly rates of return of low leverage companies' stocks.

Table 18

Results of the Z Test for Comparing High and Low Leverage Companies' Average Returns

z-Test: Two Sample for Means		
$\overline{R}_k^{LFL}$	$\overline{R}_j^{HFL}$	
Mean	2.29%	2.28%
Known Variance	0.015%	0.070%
Observations	200	200
Hypothesized Mean Differenc	0	
z	0.004	
P(Z<=z) one-tail	49.80%	
z Critical one-tail	1.64	
P(Z<=z) two-tail	99.60%	
z Critical two-tail	1.96	

The results obtained so far for test of hypothesis three, however, does not necessarily refute the proposition that average rates of return of high leverage companies are greater than average rates of return of low leverage companies. This is because both subsamples HFL and LFL contained banks and financial institutions. Banks and financial institutions typically carry a high leverage in their books due to the nature of their business, while rates of return of their stocks are on the lower side because of their conservative business nature. But, main portion of the leverage in the books of banks and financial institutions is in the form of clients' time deposits which is part of their business operations and as such is not the same thing as the leverage in nonblank corporations.

Therefore, hypothesis three was retested with banks and financial institutions being excluded from the subsamples. The result of the *z* test appears in Table 19 below:

Results of the Z Test for Comparing High and Low Financial Leverage Companies' Average Returns (Banks and Financial Institutions Excluded)

Table 19

z-Test: Two Sample for Means	$\overline{R}_k^{LFL}$	$\overline{R}_j^{HFL}$
Mean	2.16%	2.68%
Known Variance	0.02%	0.08%
Observations	176	181
Hypothesized Mean Differenc	0	
z	-2.20	
P(Z<=z) one-tail	1.39%	
z Critical one-tail	1.64	
P(Z<=z) two-tail	2.78%	
z Critical two-tail	1.96	

The results in Table 19 are quite different from those in Table 18 where banks and financial institutions were included in the subsamples. As can be seen in Table 19 the mean for the average monthly returns of low financial leverage companies over the study period was 2.16% with monthly variance of 0.02% and the mean for the average monthly returns of high financial leverage companies over the study period was 2.68% with variance of 0.08%. These combinations of mean-variance lead to the *z* statistic value of 2.2 which gives the *p* value of 1.39% for the one-tail *z* test. Therefore, at 1.40% level of significance the null hypothesis is rejected and the alternate hypothesis is accepted, meaning that excluding banks and financial institutions the average monthly rates of return of high leverage companies' stocks is higher than the average monthly rates of return of low leverage companies' stocks.

Hypothesis Four

The purpose in hypothesis four is to verify the effect of a company's operating leverage on the rate of return of the company's stock. A company's operating leverage is measured by the ratio of its fixed costs to its total costs during a period of operation. However, according to the Generally Accepted Accounting Principles (GAAP) companies are not required to report their fixed costs versus variable costs in their financial statements available to the public. Such data are companies' internal information for management control and decision making and are not publicly available. However, major portion of fixed costs in the companies' operations is due to fixed assets. Therefore, in this study the ratio of fixed assets to total assets is calculated from the companies' balance sheets and is taken as the measure of operating leverage. In hypothesis three it is intended to find out if the nonsystematic risk factor, *operating leverage*, has effects on the stocks' average rates of return. Here, the null and alternate hypotheses are stipulated on the basis of corporate finance proposition that investors regard high operating leverage companies to be more risky than low operating leverage companies because high operating leverage companies face more *business risk* than low operating leverage companies. If this is true then the rates of return of stocks of high operating leverage companies should be on average greater than the rates of return of stocks of low operating leverage companies. This proposition can be tested through the following null and alternate hypotheses:

$$H_0 : \mu(\overline{R}_j^{HOL}) \leq \mu(\overline{R}_k^{LOL})$$
$$H_1 : \mu(\overline{R}_j^{HOL}) > \mu(\overline{R}_k^{LOL})$$

where, $\overline{R}_j^{HOL}$ is average monthly rate of return of the high operating leverage company *j* over the study period (average rate of return over 120 months), $\overline{R}_k^{LOL}$ is average monthly rate of return of low operating leverage company *k* over the study period, $\mu(\overline{R}_j^{HOL})$ is the mean of all high operating leverage companies average rates of return, and $\mu(\overline{R}_j^{LOL})$ is the mean of all low operating leverage companies average rates of return.

Testing of Hypothesis Four

The first step taken in testing of hypothesis four was to measure the operating leverage of companies in the sampling frame, averaged over the study period. The companies' operating leverage were measured by calculating the ratio of the companies' fixed assets to total assets at the beginning of the study period (Dec 31, 1994) and at the end of the study period (Dec 31, 2004) and then averaging the two. An example of the procedure for measuring a company's average operating leverage during the study period is illustrated in Table 20 below.

Table 20

Operating Leverage Measurement for Hewlett Packard Company (HPQ)

	Beginning of period	**End of period**	**Average**
Fixed assets ($m)	6594	33669	
Total assets book value ($m)	18503	74381	
Operating leverage	35.64%	45.27%	**40.45%**

To prepare for testing of hypothesis four, the following procedures were taken (a) tables similar to Table 20 were worked out for every company in the sampling frame, (b) companies in the sampling frame were ordered on the basis of average operating leverage form the highest to the lowest, (c) from the list of companies with average operating leverage above the median operating leverage, 200 companies were randomly selected and were designated as the high operating leverage (HOL) subsample, (d) from the list of companies with average operating leverage below the median operating leverage, 200 companies were randomly selected and were designated as the low operating leverage (LOL) subsample, (e) in each subsample, average monthly rate of returns of every company's stock were calculated for the sample period (120 months) and the results were classified under each subsample. A portion of HOL versus LOL companies' monthly returns is exhibited in Table 21 below.

Table 21

Average Monthly Rates of Return of Stocks of High Operating Leverage Companies versus those of Low Operating Leverage Companies (Portion)

Average monthly returns(%) Low operating leverage companies $\overline{R}_k^{LOL}$	Average monthly returns(%) High operating leverage companies $\overline{R}_j^{HOL}$
1.8	5.21
1.98	0.8
1.49	1.84
1.47	2.18
2.16	1.36
2.63	1.68
2.01	2.3
1.85	1.78
3	2.8
1.99	1.49
2.37	1.3
1.38	4.69
0.88	1.09
1.63	4.13
1.73	3.08
1.83	1.95

The *z* test technique was applied to the data in Table 21 to verify if the mean monthly rate of return of stocks of low operating leverage companies is greater than that of high operating leverage companies. The result of the *z* test is illustrated in Table 22 below:

Table 22

Results of the Z Test for Comparing High and Low Operating Leverage Companies' Average Returns

z-Test: Two Sample for Means

	$\overline{R}_k^{LOL}$	$\overline{R}_j^{HOL}$
Mean	2.23%	2.27%
Known Variance	0.014%	0.073%
Observations	200	200
Hypothesized Mean Differen	0	
z	-0.02	
P(Z<=z) one-tail	49.20%	
z Critical one-tail	1.64	
P(Z<=z) two-tail	99.40%	
z Critical two-tail	1.96	

The results in Table 22 indicate that the mean for the average monthly returns of low operating leverage companies over the study period was 2.23% with monthly variance of 0.014% and the mean for the average monthly returns of high operating leverage companies over the study period was 2.27% with monthly variance of 0.073%. These combinations of mean-variance lead to the *z* statistic value of -0.02 which gives the *p* value of 49.20% for the one-tail *z* test. Therefore the null hypothesis is not rejected and the alternate hypothesis is rejected, meaning that based on the sample results the average monthly rates of return of high operating leverage companies' stocks is *not* higher than the average monthly rates of return of low operating leverage companies' stocks. Because of this finding, hypothesis five is modified and the operating leverage is dropped from the regression equations proposed in hypothesis five.

Hypothesis Five

Hypothesis five is this author's proposed contribution to the field. It is a multifactor model and posits that (a) the expected rate of return of every stock is linearly dependent on five risk factors, the market return, the market implied volatility, the company's asset size, the company's financial leverage, and the company's operating leverage and (b) expected rates of return across cross-section of stocks are linearly dependent on the coefficients of risk factors, the betas, estimated in part (a). However, in testing of hypothesis four it was found that the average rates of return of high operating leverage companies is statistically not different from those of low operating leverage companies. Therefore, the original proposition in hypothesis five is modified and the operating leverage is excluded from the regression equations embodied in hypothesis five. Thus, part (a) of hypothesis five is tested through the regression equation:

$$R_{jt} - R_{ft} = \alpha_j + \beta_j^M (R_{tM} - R_{ft}) + \beta_j^{IV} (IV_{t-1}) + \beta_j^S (SLL_t) + \beta_j^{FL} (HFLLF_t) + e_{jt} \qquad (44)$$

where,

$R_{jt} - R_{ft}$ = excess rate of return of stock *j* during the month *t*;

$R_{tM} - R_{ft}$ = excess rate of return of the market (S&P500) during the month *t* and represents the market risk factor which is a systematic risk,

IV_{t-1} =the implied market volatility at the end of month (*t-1*) measured by VIX value at the end of month (*t-1*) and is a systematic risk factor. The one month time-lag is incorporated in the model because option contracts are forward looking financial instruments and the implied market volatility derived from options' current prices represents expectation of future volatilities. The values of this variable was fed into the regression equation as a month -to -month percentage change to be commensurate with the other variables that all in terms of rate of change,

SLL_t = the difference between average rate of return of small companies and average rate of return of large companies during the month *t*. This is a nonsystematic risk factor and is measured by finding the simple average rate of returns of all companies in the LAS subgroup during period *t* and subtracting it from simple average rate of returns of all companies in the SAS subgroup during period *t.* In other words, $SLL_t = \overline{R}_t^{SAS} - \overline{R}_t^{LAS}$,

$HFLLF_t$: The difference between average rate of return of high financial leverage companies and average rate of return of low financial leverage companies during the month *t*. This is a nonsystematic risk factor and is measured by finding the simple average rate of returns of all companies in the LFL subgroup during period *t* and subtracting it from simple average rate of returns of all companies in the HFL subgroup during period *t.* In other words, $HFLLF_t = \overline{R}_t^{HFL} - \overline{R}_t^{LFL}$,

β_j^M = the responsiveness of stock *j* rate of return to the market risk factor,

β_j^{IV} = the responsiveness of stock *j* rate of return to the implied volatility risk factor,

β_j^S = the responsiveness of stock *j* rate of return to the size risk factor, and

β_j^{FL} = the responsiveness of stock *j* rate of return to financial leverage risk factor.

Using the language of test of hypothesis, the null and alternate hypotheses for the first equation of hypothesis five would be:

$$H_0 : \alpha_j, \beta_j^M, \beta_j^{IV}, \beta_j^S, \beta_j^{FL} = 0$$
$$H_1 : \alpha_j, \beta_j^M, \beta_j^{IV}, \beta_j^S, \beta_j^{FL} \neq 0$$

If the proposition that the rate of return of every stock *j* is determined by the four specified risk factors is true then the result of the test should lead to the conclusion that for every company the regression intercept α_j is statistically not different from zero and all the β_j's are statistically different from zero.

Part (b) of hypothesis five is tested through the regression equation:

$$\overline{R_j - R_f} = \lambda_0 + \lambda_1 b_j^M + \lambda_2 b_j^{IV} + \lambda_3 b_j^S + \lambda_4 b_j^{FL} + e_j \qquad (45)$$

where,

$\overline{R_j - R_f}$ = average risk premium of stock *j* during the study period (average rate of return of stock *j* less average risk-free rate during the study period). This is the dependent variable of the regression equation,

b_j's = estimates of the β_j's found from the first regression equation, Equation 44, serving as the independent variables of the equation' and

λ's = regression coefficients of Equation 45 representing the expected values of the risk factors, that is, the expected values of market excess return, market implied volatility, asset size premium, and financial leverage premium.

Using the language of the test of hypothesis, the null and alternate hypotheses for the second part of hypothesis five can be stated as the following:

$$H_0 : \lambda_0 = 0,\ \lambda_1 = \overline{R_M - R_f},\ \lambda_2 = \overline{IV},\ \lambda_3 = \overline{SLL},\ \lambda_4 = \overline{HFLLF}$$

$$H_1 : \lambda_0 \neq 0,\ \lambda_1 \neq \overline{R_M - R_f}\ ,\ \lambda_2 \neq \overline{IV}\ ,\ \lambda_3 \neq \overline{SLL},\ \lambda_4 \neq \overline{HFLLF}$$

Testing of Hypothesis Five: First Part

The first step in testing the first part of hypothesis five was to prepare the data that should be fed in regression Equation 44. Two sets of required data, namely, monthly excess rates of return for every stock in the sample, $R_{jt} - R_{ft}$, and monthly excess rates of return for the S&P500 Stock Index, $R_{Mt} - R_{ft}$, were already prepared for testing of hypothesis one. Monthly data on the independent variable, market implied volatility, IV_t were retrieved from CBOE website.

Preparing the data for the other two independent variables of Equation 44, however, entailed some new calculations. To calculate monthly values of the variable SLL_t, monthly average rates of return of small companies less monthly average rates of return of large companies, the following steps were taken (a) for each month in the sample period monthly rates of return of all the 200 stocks in the SAS subsample were measured and the average rate of return for that month, $\overline{R}_t^{SAS}$, was calculated, (b) similarly, for each month in the sample period monthly rates of return of all the stocks in the LAS subsample were measured and the average rate of return for that month, $\overline{R}_t^{LAS}$, was calculated, and (c) for each month in the sample period the difference between $\overline{R}_t^{SAS}$ and $\overline{R}_t^{LAS}$ was calculated and used as the value of SLL_t variable in the regression Equation 44. The process to calculate $\overline{R}_t^{SAS}$ is demonstrated in Table 23 below.

Table 23

Calculating Average Monthly Rates of Return of Every Small Company (Portion)

	$\overline{R}_t^{SAS}$ Small companies' monthly average returns					
Date/Company	AACE	ABM	ABMD	KLIC	LBY	Average:
1-Dec-04	11.17%	-10.14%	1.71%	15.24%	5.92%	4.46%
1-Nov-04	2.58%	5.78%	34.10%	4.76%	16.17%	7.85%
1-Oct-04	-0.12%	3.45%	27.91%	26.37%	-2.99%	2.69%
1-Sep-04	-4.62%	12.23%	-9.14%	3.10%	-1.02%	4.43%
1-Jun-04	7.94%	2.38%	1.86%	-4.28%	8.80%	4.35%
3-May-04	-21.97%	3.00%	8.05%	16.36%	-2.11%	2.03%
3-Apr-95	0.00%	3.21%	20.99%	54.82%	3.39%	4.36%
1-Mar-95	1.36%	-1.22%	-17.35%	12.32%	2.04%	2.47%
1-Feb-95	-2.91%	-3.07%	55.32%	23.28%	16.40%	7.64%

The same procedure as shown in Table 23 was done to calculate average monthly rates of return of large companies, $\overline{R}_t^{LAS}$, and then the difference between $\overline{R}_t^{SAS}$ and $\overline{R}_t^{LAS}$ was used as the SLL_t input in the regression Equation 44.

To calculate monthly values of the variable $HFLLF_t$, monthly average rates of return of high financial leverage companies less monthly average rates of return of low financial leverage companies, the following steps were taken (a) for each month in the sample period monthly rates of return of all the 200 stocks in the HFL subsample were measured and the average rate of return for that month , $\overline{R}_t^{HFL}$, was calculated, (b) similarly, for each month in the sample period monthly rates of return of all the stocks in the LFL subsample were measured and the average rate of return for that month , $\overline{R}_t^{LFL}$, was calculated, and (c) for each month in the sample period the difference between $\overline{R}_t^{HFL}$

and $\overline{R}_t^{LFL}$ was calculated and used as the value of $HFLLF_t$ variable in the regression Equation 44. The process to calculate $\overline{R}_t^{HFL}$ is demonstrated in *Table 24* below:

Table 24

Calculating Average Monthly Rate of Return of all High Financial Leverage Companies (portion)

	High financial leverage companies' monthly average returns $\overline{R}_t$					
Date/Company	ACI	ACS	ADPT	MAY	MOT	Average:
1-Dec-04	-6.76%	1.71%	-2.69%	4.54%	0.00%	4.60%
1-Nov-04	17.47%	8.49%	0.00%	8.78%	11.60%	8.36%
1-Oct-04	-8.36%	-2.01%	2.63%	1.67%	-4.36%	2.13%
1-Sep-04	10.38%	2.47%	8.88%	4.59%	12.01%	4.68%
2-Aug-04	-4.57%	4.68%	-6.81%	-6.73%	1.34%	0.00%
1-Jul-04	-7.71%	-1.96%	-11.47%	-3.49%	-12.67%	-2.93%
1-Jun-04	12.87%	6.26%	3.17%	-4.07%	-7.54%	5.18%
3-May-04	6.17%	2.72%	5.26%	-6.15%	8.36%	1.99%
1-May-95	-2.26%	1.91%	-5.86%	9.48%	5.23%	5.11%
3-Apr-95	-5.58%	-6.84%	-3.04%	-2.05%	4.17%	3.28%
1-Mar-95	7.82%	20.03%	0.00%	1.37%	-4.86%	3.52%
1-Feb-95	-3.53%	2.53%	20.56%	4.76%	-2.92%	3.72%

The same procedure as shown in Table 24 was done to calculate average monthly rates of return of low financial leverage companies, $\overline{R}_t^{LFL}$, and then the difference between $\overline{R}_t^{HFL}$ and $\overline{R}_t^{LFL}$ was calculated and used as the $HFLLF_t$ input in the regression Equation 44.

To estimate the regression coefficients of Equation 44 and test for statistical significance of the estimated coefficients, for every company in the sample time-series data for the variables of the regression were organized in Excel worksheets. An example of this task for the case of Yahoo Corporation (ticker symbol YHOO) is partially illustrated in Table25 below:

Table 25

Time Series Data on Monthly Excess Returns of Yahoo Corporation's Stock and Monthly Values of Four Risk Factors, January 1995 to December 2004

Date	YHOO monthly excess returns $R_{jt} - R_{ft}$	S&P 500 monthly excess returns $R_{Mt} - R_{ft}$	Market implied volatility $IV_{(t-1)}$	Small less large companies' returns SLL_t	High less low financial leverage companies' returns HFLLF
1-Dec-04	-0.026%	3.061%	-20.28%	0.88%	0.28%
1-Nov-04	3.776%	3.684%	27.61%	1.58%	-0.92%
1-Oct-04	6.575%	1.252%	-14.49%	0.30%	-0.19%
1-Sep-04	18.801%	0.796%	-2.99%	1.48%	0.45%
2-Aug-04	-7.560%	0.104%	1.12%	-2.12%	-0.29%
1-Jul-04	-15.498%	-3.542%	-6.75%	-3.06%	1.90%
1-Aug-95	-0.450%	-0.482%	17.20%	-1.26%	-0.29%
3-Jul-95	-0.452%	2.726%	-5.24%	3.59%	-3.88%
1-Jun-95	-0.456%	1.672%	0.58%	0.50%	1.26%
1-May-95	-0.464%	3.167%	-10.07%	-2.38%	-2.13%
3-Apr-95	-0.471%	2.325%	15.88%	1.54%	-0.21%
1-Mar-95	-0.478%	2.255%	-0.68%	-0.04%	1.84%
1-Feb-95	-0.481%	3.127%	-17.68%	2.93%	-2.38%

Using the regression feature of Excel, the data organized in Table 25 were then used to solve the regression Equation 44 and to find the regression coefficients and their statistical significance for every stock in the sample. Regression results for the case of Yahoo Corporation (YHOO) are exhibited in Table 26 below. As the regression results in Table 26 indicate, for the case of YHOO stock, the *p* value for the multiple correlation is zero (in the ANOVA table) which suggests that the dependent variable is correlated to the independent variables taken together, that is, the multiple correlation coefficient is statistically different from zero. The regression intercept value is -0.116 with *p* value of 5.4% which does not reject the null hypothesis that α_j for YHOO is statistically equal to zero. But the p value for all other regression coefficients are below 2% leading to rejection of the null hypothesis that any of the β_j's are statistically equal to zero. Therefore, in the case of YHOO stock all the four risk factors, market excess return, implied volatility, asset size, and leverage are significant factors in determining the rate of return of YHOO stock. The R-square value for YHOO is 45.35% implying that about half of variations in the monthly rate of returns of YHOO are due to variations in the four risk factors of the model.

Table 26

Multiple Regression Output: YHOO Monthly Excess Returns versus Four Risk Factors

SUMMARY OUTPUT

Regression Statistics	
Multiple R	67.35%
R Square	45.35%
Adjusted R Square	43.44%
Standard Error	18.02%
Observations	119

ANOVA

	df	SS	MS	F	P-value
Regression	4	3.07	0.77	23.65	0.00%
Residual	114	3.70	0.03		
Total	118	6.78			

	Coefficients	Standard Error	t Stat	P-value
Intercept	-0.116	0.060	-1.95	5.395%
S&P monthly excess returns $\boldsymbol{R_{Mt}-R_{ft}}$	2.883	0.377	7.64	0.000%
Implied volatility $\boldsymbol{IV_{(t-1)}}$	0.117	0.048	2.42	1.708%
Small less large $\boldsymbol{SLL_{(t)}}$	1.429	0.451	3.17	0.197%
High less low financial leverage $\boldsymbol{HFLLF_{(t}}$	-1.489	0.613	-2.43	1.671%

To test the first part of hypothesis one as exemplified in Equation 44, tables similar to Table 25 and Table 26 were constructed for all the 855 stocks in the sample and for each of them regression results were recorded in tables similar to Table 26. The results were then summarized on the basis of frequency distribution of the α_j and β_j's statistical significance and for different ranges of R-Square values. The results for α_j and β_j's statistical significances are reported in Table 27 below:

Table 27

Summary of Regression Results for the 855 Stocks in the Sample: Statistical Significance of Regression Coefficients.

Summary of regression coefficients	At 1% level	At 5% level	At 10% level
Percentage of α_j 's significantly not different from zero	98%	96%	86%
Percentage of $\beta^M{}_j$'s significantly different from zero	55%	73%	78%
Percentage of $\beta^{IV}{}_j$'s significantly different from zero	66%	75%	88%
Percentage of $\beta^S{}_j$'s significantly different from zero	75%	89%	95%
Percentage of $\beta^{FL}{}_j$'s significantly different from zero	70%	76%	83%

Summary results for percentage distribution of R-square values for different ranges of R-squares are shown in Table 28 below:

Table 28

Summary of Regression Results for 855 Stocks: Percentage Distribution of Adjusted R-Square Values

	Summary of R-square results
Adjusted R-sqaure range	Percentage of stocks in the range
0-10%	5%
10%-20%	9%
20%-30%	16%
30%-40%	19%
40%-50%	36%
50%-60%	9%
60%-70%	6%
Over 70%	0%
Average adjusted R-square	37.80%
Median adjusted R-square	41.60%

Testing of Hypothesis Five: Second Part

The second part of hypothesis five, as expressed in regression Equation 45 and the related null and alternate hypotheses regards the average monthly excess returns of the stocks in the sample, the $\overline{R_j - R_f}$'s , as the dependent variable with the estimates of the betas, the b_j 's, obtained form the regression equation of the first part of hypothesis five as the independent variables. To test the second part of hypothesis five, average monthly rates of return for every stock in the sample, the $\overline{R_j}$'s, and average monthly risk-free rates, the $\overline{R_f}$, over the sample period (120 months) were calculated and together with the estimated b_j 's were organized in an Excel worksheet a portion of which is illustrated in Table 29 below:

Table 29

Stocks' Average Monthly Returns versus their Betas (partial)

Stock symbol	$\overline{R_j}$ (%)	$\overline{R_f}$ (%)	$\overline{R_j - R_f}$ (%)	b_j^M	b_j^{IV}	b_j^S	b_j^{FL}
AACE	2.34	0.32	2.02	0.549	-0.225	0.551	-0.844
ACI	1.4	0.32	1.08	0.181	0.217	-0.281	0.066
BRL	3.19	0.32	2.87	0.653	0.009	0.298	-0.712
BSET	0.63	0.32	0.31	0.686	0.036	0.118	0.127
CEPH	3.72	0.32	3.4	0.973	0.068	1.128	-0.759
CHUX	1.34	0.32	1.02	0.607	0.065	-0.406	-0.140
WGOV	2.03	0.32	1.71	0.772	0.016	0.369	0.189
WON	2.17	0.32	1.85	1.057	0.378	0.263	-0.569
XRIT	0.69	0.32	0.37	-0.028	0.092	0.874	-0.927
YANB	1.82	0.32	1.5	0.567	-0.040	-0.122	0.040
YHOO	6.07	0.32	5.75	2.883	0.117	1.429	-1.489

The data in Table 29 were then used to solve the regression Equation 45 and thus test the second part of hypothesis five. As was pointed out before, part two of hypothesis five posits that (a) average or expected monthly rates of return of stocks in excess of the average (expected) monthly risk-free are linearly dependent on their betas, with the betas being the responsiveness of the stocks' rates of return to the four risk factors studied in part one of hypothesis five (b) the intercept coefficient of the regression passes through the origin implying that when no risk factor is present the expected rate of return of every stock is equal to expected risk-free rate , and (c) other coefficients of the regression Equation 45 are equal to the expected (average) values of the relevant risk factors. The results of the regression conducted through Microsoft Excel appear in Table 30 below:

Table 30

Regression Output: Stocks' Average (expected) Monthly Returns versus their Betas

SUMMARY OUTPUT

Regression Statistics	
Multiple R	0.8975
R Square	80.56%
Adjusted R Square	80.47%
Standard Error	0.63%
Observations	855

ANOVA

	df	*SS (%)*	*MS (%)*	*F*	*P-value*
Regression	4	12.68	3.17	792.50	0%
Residual	850	3.06	0.004		
Total	854	15.74			

	Coefficients	*Standard Error*	*t Stat*	*P-value*
a_j (intercept)	0.436%	0.082%	5.32	0.00%
b_j^M	0.636%	0.171%	3.74	0.02%
b_j^{IV}	1.17%	0.34%	3.44	0.04%
b_j^S	0.63%	0.16%	3.94	0.00%
b_j^{FL}	0.41%	0.15%	2.73	0.64%

According to the regression output in Table 30, the coefficient of multiple correlations is 0.89 with a *p* value of 0.00% implying that the dependent variable is strongly correlated with the combination of independent variables. As for the significance of coefficients of individual independent variables (the betas), as can be seen in the regression output, all the regression coefficients including the intercept have *p* values of less than 1% which makes them to be statistically different from zero.

During the study period (120 months) average monthly risk-free rate ($\overline{R_f}$) was 0 .32% , average monthly return on S&P 500 Stock Index ($\overline{R_M}$) was 0.90% (making $\overline{R_M - R_f} =$ 0.58%), average change in monthly implied volatility was 1.21% ($\overline{IV}$ =

1.21%), average monthly small company less large company rate of return was 0.71% ($\overline{SLL}$ = 0.71%, result from test of hypothesis two), and average monthly high financial leverage less low financial leverage company rate of return was 0.18% ($\overline{HFLLF}$ = 0.52%, result from test of hypothesis three) . This information and the information provided in Table 30 are combined in Table 31 below to be utilized for testing of the second part of hypothesis five.

Table 31

Regression Results for Test of Hypothesis Five, Second Part

	λ_0	λ_1	λ_2	λ_3	λ_4
Coefficient	0.436%	0.63%	1.17%	0.63%	0.41%
Hypothetical value	0	0.58%	1.21%	0.71%	0.52%
Standard error	0.082%	0.17%	0.34%	0.16%	0.15%
t Statistic	5.32	0.29	0.117	0.5	0.73
P-value	0.00%	77.18%	45.34%	61.72%	46.56%
Adjusted R-square = 80.47%					

As per the information in Table 31, regression estimate for λ_0 is 0.436% with standard error of 0.082%. This means that the estimate of λ_0 is away from its hypothesized value of zero by 5.32 standard error (t = 5.32, *p* value = 0.00%). Therefore, the null hypothesis that $\lambda_0 = 0$ is rejected. The regression estimate for λ_1 is 0.63% with standard error of 0.17% which makes it to be different from its hypothesized value of 0.58% by only 0.29 standard error of the estimate (t = 0.29, *p* value = 77.18%). Thus, the hypothesis that $\lambda_1 = \overline{R_M - R_f}$ = 0.58% is not rejected. The regression estimate for $\boldsymbol{\lambda_2}$ is 1.17% with standard error of 0.34% which makes it to be different from its hypothesized value of 1.21% by only 0.117 standard error of the estimate (t = 0.117, *p* value = 45.34%). Thus, the hypothesis that $\lambda_2 = \overline{IV}$ = 1.21% is not rejected. The regression estimate for λ_3 is 0.63% with standard error of 0.16% which makes it to be different from its hypothesized value of 0.71% by only 0.5 standard error of the estimate (t = 0.5, *p* value = 61.72%). Thus, the hypothesis that λ_3 = $\overline{SLL}$ = 0.71% is not rejected. The regression estimate for $\boldsymbol{\lambda_4}$ is 0.41% with standard error of 0.15% which makes it to be different from its hypothesized value of 0.52% by only 0.73 standard error of the estimate (t = 0.73, *p* value = 46.56%). Thus, the hypothesis that λ_4 = $\overline{HFLLF}$ = 0.52% is not rejected.

Test for Autocorrelation

When a regression equation involves time-series data it might be possible to have the problem of autocorrelation (serial correlation), in which case the estimates of the regression coefficients obtained through the least square method will be biased. Autocorrelation is a correlation between the values of a variable with the values of the same variable lagged some time period back and manifests itself in the form of correlation between the residuals (the error terms) of the regression model.

Existence of first -order autocorrelation is tested through calculating the Durbin-Watson (DW) test statistic and checking it against the critical values of the statistic. The formula for calculating the DW test statistic is given by:

$$d = \frac{\sum_{i=2}^{n}(e_i - e_{i-1})^2}{\sum_{i=1}^{n} e_i^2}$$ where, n is the number of observation and e is the regression residual term. The critical values of the statistic are related to the number of independent variables and the number of observations and are provided in statistical tables for various significance levels. There are two critical values for DW test statistic, lower value d_l and upper value d_u. Interpretation of different values of the DW test statistic with regard to autocorrelation is as shown in Figure: 7

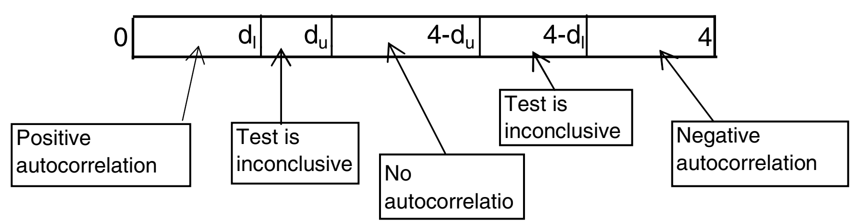

Figure 7. Critical regions for the Durbin-Watson test.

In hypothesis five the regression Equation 44 involves time-series data in which monthly rates of return of the stocks in the sample were regressed against monthly values of the four risk factors for the sample period, January 1995 to December 2004. To test for autocorrelation in regression Equation 44, first all the stocks in the sample were combined into an equally weighted portfolio and then monthly rates of return of the resulting portfolio were regressed against the four risk factors. The value of the Durbin-Watson test statistic for the resulting regression was calculated to be 1.83. According to statistical tables, the critical values of the DW test at 1% level of significance and for four independent variable for 120 observations are d_l = 1.46 and d_u = 1.63 (Aczel, 2002, pp.

854-855). The number 1.83 falls inside the *no autocorrelation* region, because it is between 1.63 and (4-1.63 = 2.37). Therefore, it is concluded that on average the residual terms in the regression Equation 44 do not exhibit first-order autocorrelation.

Test for higher order autocorrelations were also conducted using Ljung-Box Q Statistics in MINITAB Software. The results for up to six periods of time-lags are demonstrated in Figure 8 below which confirms lack of autocorrelation.

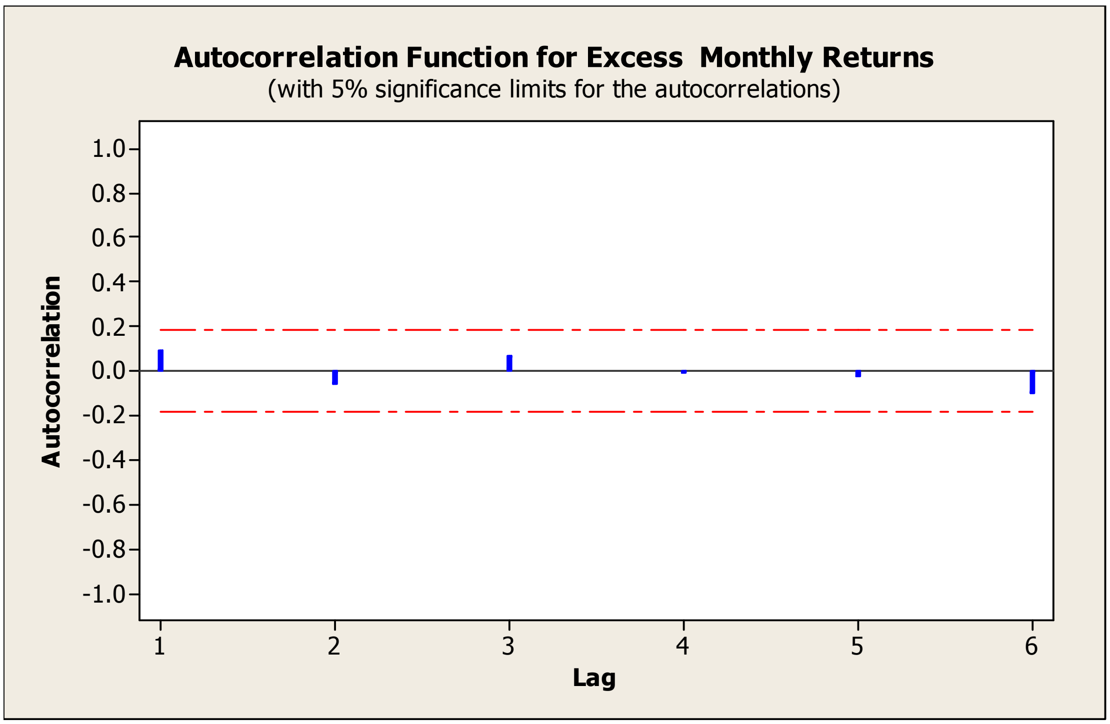

Figure 8. MINITAB output for excess monthly returns' autocorrelation.

Test for Multicollinearity

Multicollineaity is a potential problem in dealing with multiple regressions. It occurs when the independent variables of the model are *highly* correlated which results in high variance of the regression estimators (high standard errors) and, therefore, might lead to incorrect conclusions as to which independent variables are statistically significant. In case of perfect multicollinearity, that is when, some of the independent variables are perfectly correlated; the regression algorithm totally breaks down. However, as in real life, especially in the area of economics and finance, usually the variables of interest are interrelated; the seriousness of multicollinearity in regression analysis is a problem of degree. The common rule of practice is that correlations among the independent variables between -0.70 and +0.70 do not cause difficulties and are not regarded as high multicollinearity (Lind, Marchal, & Mason, 2002, pp. 514-515). The degree of multicollinearity is determined by constructing the correlation matrix of the independent variables and observing which variables are highly correlated. Multicollinearity is also examined by calculating the variance inflation factor (VIF) for every independent variable. The VIF measures the variance of a regression estimator as the ratio of what it would have been if the related independent variable was not collinear with any of the other independent variables. The common rule of practice is that a VIF value of below 5 for an estimator does not cause difficulties and the collinearity of the related independent variable is not serious.

Both of the regression equations in hypothesis five involve multiple independent variables and need to be checked for multicollinearity. In the first part of hypothesis five the independent variables in regression Equation 44 are the market risk factor, $R_{Mt} - R_{ft}$, the implied volatility risk factor, IV_t, the size risk factor, SLL_t, and the financial leverage risk factor, $HFLLF_t$. The correlation matrix as well as the VIF values for these independent variables was calculated using their monthly values for the study period (120 months) and the result is shown in Table 32 below:

Table 32

Correlation Matrix for the Independent Variables in Hypothesis Five, First Part

	$R_{Mt}-R_{ft}$	***IV***	***LLS***	***HFLLF***
$R_{Mt}-R_{ft}$	1			
IV	-0.12	1.00		
LLS	0.09	-0.10	1.00	
HFLLF	-0.15	-0.07	0.34	1
VIF	1.1	1.0	1.2	1.2

As can be seen from the results in Table 32, none of the independent variables in Equation 44 are highly correlated. The highest correlation is between $\boldsymbol{SLL_t}$ and $\boldsymbol{HFLLF_t}$ with a correlation coefficient of 0.34 which is far below the 0.70 threshold. Also all the VIF values are below 5 and quite small.

The regression model for the second part of hypothesis five, as expressed through regression Equation 45, also contains four independent variables. These are the four betas which were estimated for every stock in the sample through regression Equation 44. The estimated betas were b^M, for the market effect, b^{IV}, for the implied volatility effect, b^S, for the size effect, and b^{FL}, for the financial leverage effect. The correlation matrix and the VIF values for these independent variables were calculated using the beta estimates for all the stocks in the sample and the result is shown in Table 33 below:

Table 33

Correlation Matrix for the Independent Variables in Hypothesis Five, Second Part

	B^{M}	B^{IV}	B^{S}	B^{FL}
B^{M}	1			
B^{IV}	0.55	1.00		
B^{S}	0.48	0.54	1.00	
B^{FL}	0.03	0.01	-0.54	1.00
VIF	1.4	1.3	1.6	1.5

The results in Table 33 indicate that multicollinearity is not a problem for regression Equation 45 as all the correlation coefficients are within the permitted threshold region of -0.70 to +0.70 and the VIF values are all below 5. In fact, lack of multicollinearity in regression Equation 45 can be substantiated by looking at the regression output in Table 30. As was mentioned before the most serious outcome of multicollinearity in regression analysis is large standard errors for the estimates of the regression coefficients. Large standard error produces small *t* statistic and consequentially large *p* values. The regression output in Table 30, not only does not show large *p* values but in fact all the *p* values are extremely small indicating that multicollinearity is not present.

Summary

In this chapter all the five hypotheses posited in the study were tested against relevant empirical data. Despite most tests of the CAPM that concentrate on portfolios of stocks, the focus of this study was on validity of CAPM for individual stocks, as theoretically this is what CAPM claims. All the five hypotheses stipulated and tested were about the relationships between rates of return of individual stocks and specific risk factors. Data were collected on a monthly basis. The period for which data were collected and analyzed was from January 1, 1995 to December 31, 1994.

In hypothesis one both tenets of the standard CAPM (single-factor model) was tested. To test the first part of standard CAPM, time series data on monthly excess rates of return of a sample of 855 stocks were regressed against monthly excess rates of return of S&P500 stock index for the period January 1, 1995 to December 31, 1994. As was reported in Table 8 in this chapter, at 1% level of significance 99% of the socks in the sample had a regression intercept not significantly different from zero, which is a good result in conformity with the standard CAPM prediction. The results for the betas, the slope coefficients, were not as good as the results for the intercepts, as at 1% level only 56% of the estimates for betas were significantly different from zero, though at 10% level about 77% of the beta estimates turned out to be significantly different from zero. In other words, at 1% level of significance variations in the market rates of return was correlated to variations in the rates of return of only 56% of the stocks in the sample. The results for distribution of the adjusted R-squares, as was reported in Table 9 were even less in support of the standard CAPM hypothesis. For the 855 regression equations studied, the average adjusted R-square was 9.78%. And the median adjusted R-square was 6.60% implying that for 50% of the stocks in the sample the market return could account for only 6.60% of the variations in the stocks' monthly rates of return. Moreover, as was shown in Table 9, for 94% of the stocks the adjusted R-squares were below 30%, no stock had an adjusted R-square above 60%, and only 1% of the stocks in the sample had adjusted R-square of above 50%.

To test the second part of standard CAPM, the 855 beta estimates with the corresponding variances of the regression residual estimates computed in the first part were regressed against stocks' monthly excess rates of return averaged over the whole study period. The result of the second pass-regression strongly supports one part of the CAPM second's tenets, but at the same time strongly rejects the other two parts. As is reported in Table 12 of this chapter the estimate for the intercept of the second-pass regression equation comes out to be significantly different from zero, in contrast to CAPM prediction that it must be zero. The estimate for the coefficient of the independent variable beta is equal to 59% just 1% above the monthly average rate of return for S&P500 Stock Index over the study period; a strong support for the CAPM proposition that expected excess rates of returns of stocks are linearly related to their betas by expected excess market rate of return. However, the result of the test of the second-pass regression showed a strong correlation between stock's expected excess rates of return and their nonsystematic risks, as measured by the variance of the residual terms from the first-pass regression equations. This result strongly rejects the CAPM proposition that nonsystematic risk has no role in determining stock's expected rates of return.

In hypothesis two the proposition that expected rate of return of small companies is larger than expected rate of return of large companies was tested. For testing of this hypothesis, monthly average rates of return of 200 randomly selected small companies were compared with monthly average rates of return of 200 randomly selected large companies. The results of the z-test as exhibited in Table 15 support this idea and it turns out that on average monthly rate of return of small companies' stocks is significantly greater than monthly rate of return of large companies' stocks, and this significant monthly difference is 0.71%.

In hypothesis three the proposition that expected rate of return of high financial leverage companies is larger than expected rate of return of low financial leverage companies was tested. For testing of this hypothesis, monthly average rates of return of 200 randomly selected high financial leverage companies were compared with monthly average rates of return of 200 randomly selected low financial leverage companies. The results of the z-test as exhibited in Table 19 support this idea when banks and financial institutions are excluded from the sample and it turns out that on average monthly rate of return of high financial leverage companies' stocks is significantly greater than monthly rate of return of low financial leverage companies' stocks, and this significant monthly difference is 0.52%.

In hypothesis four the proposition that expected rate of return of high operating leverage companies is larger than expected rate of return of low operating leverage companies was tested. For testing of this hypothesis, monthly average rates of return of 200 randomly selected high operating leverage companies were compared with monthly average rates of return of 200 randomly selected low operating leverage companies. The results of the *z* test as exhibited in Table 22 did not support this idea. Monthly average rate of return of high operating leverage companies was 2.27% which was not significantly different from 2.23% monthly average rate of return of low operating leverage companies.

In hypothesis five the author's proposed multifactor model was tested. According to the proposed multifactor model (a) expected rate of return of every stock is linearly dependent on two systematic risk factors; the market return, the market implied volatility and three nonsystematic risk factors; the company's asset size, the company's financial leverage, and the company's operating leverage and (b) expected rates of return across cross-section of stocks are linearly dependent on the coefficients of risk factors, the betas, estimated in part (a). However, because of the finding in testing of hypothesis four that the average rates of return of high operating leverage companies is statistically not different from those of low operating leverage companies, the original proposition in hypothesis five was modified and the operating leverage was excluded from the regression equations embodied in hypothesis five.

To test the first part of the multifactor model, time series data for the period January 1, 1995 to December 31, 1994 on monthly excess rates of return for the stock of every company in the sample (855 stocks) were regressed against monthly excess rates of return of S&P500 Stock Index, monthly change in implied volatility lagged one month, monthly difference between rates of return of small companies and large companies, and monthly difference between rates of return of high financial leverage companies and low financial leverage companies. As was reported in Table 27 in this chapter, at 1% level of significance 98% of the socks in the sample had a regression intercept estimate not significantly different from zero, which is a good result in conformity with expectation from the model. As for the estimates of the risk factor coefficients it was found that at 1% level the market beta estimates for 55% of the stocks were significantly different from zero, implied volatility beta estimates for 68% of the stocks were significantly different from zero, size beta estimates for 66% of the stocks were significantly different from zero, and financial leverage beta estimates for 70% of the stocks were significantly different from zero. At 10 percent significance level, however, for over 78% of the stocks in the sample all the four beta estimates were statistically different from zero. In other words, at 10% level of significance variations in the rates of return of over 78% of the stocks in the sample were linearly related to variations in the four hypothesized risk factors. Regarding the adjusted R-square measure for the individual stocks in the sample, as was reported in Table 28, for the 855 regression equations studied, the average adjusted R-square was 37.80% and the median adjusted R-square was 41.60% implying that for 50% of the stocks in the sample the four specified risk factors could account for 41.60% of the variations in the stocks' monthly rates of return. Moreover, for 94% of the stocks the adjusted R-squares were above 50%, the adjusted R-square for 6% of the stocks in the sample were above 60%, and there was no stock with adjusted R-square above 70%.

To test the second part of hypothesis five, the four beta estimates for the 855 stocks in the sample were regressed against stocks' monthly excess rates of return averaged over the whole study period. The idea was to test if differences in the expected rates of return in a cross section of stocks are due to their different sensitivities to the four specified risk factors. The results of the second pass-regression came out to be in strong support of the idea. As was reported in Table 30, the estimate for the regression intercept was significantly different from zero, not as expected, but the estimates for all the beta coefficients were significantly different from zero, as expected. Moreover, according to the results reported in Table 31 it was found that the estimates for the coefficients of the betas were not significantly different from their hypothesized values.

CHAPTER 5

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

Summary and Findings

This study was done to evaluate the validity of the standard single-factor CAPM, to find out about its strengths and shortcomings, and to come out with and test a model, the multifactor model, which provides more valid explanations for variations of stocks' rates of return in the U.S. stock market. The study covered the time period of January 1, 1995 to December 31, 2004 as not many empirical studies of CAPM have covered this time period. Five hypotheses were postulated and tested against monthly data for the specified time period.

Hypothesis one was to test the validity of standard single factor CAPM. In testing of the first proposition of standard CAPM it was found that for 99% of the stocks in the sample the estimate of the regression intercept was not statistically different from zero at 1% level of significance; in conformity with the first proposition of standard CAPM. Furthermore, in testing the first proposition of the CAPM it was found that for 56% of the stocks the slope estimate was significantly different from zero, the average adjusted R-square was 9.78%, and median adjusted R-square was 6.6%. In testing of the second proposition of the standard single factor CAPM it was found that the intercept of the regression line, the intercept of the SML, was significantly different from zero, the slope of the line (coefficient of the beta) was not significantly different from average excess monthly rate of return of the market over the study period, and the coefficient of the nonsystematic risk was significantly different from zero.

Hypothesis two was to find out if company's size, as measured by the market value of the company's total assets, affects the rate of return expected from the stock. The results of the *z* test for two population means confirmed the idea that stocks of small companies on average realize higher rate of return than stocks of large companies. Hypothesis three was to verify if financial leverage is a significant factor in determining the expected rates of return of stocks. The results of the *z* test for two population means confirmed the idea that stocks of high financial leverage companies on average realize higher rate of return than stocks of low financial leverage companies. Hypothesis four was to find out if operating leverage is a significant factor in determining the expected rates of return of stocks. The results of the *z* for two population means did not confirm the idea that stocks of high operating leverage companies on average realize higher rate of return than stocks of low operating leverage companies.

Hypothesis five was the author's proposition to address the shortcomings of the standard CAPM, as verified in testing of hypothesis one, and incorporate the findings from testing of hypothesis two, three, and four. In testing of the first proposition of hypothesis five it was found that for 98% of the stocks in the sample the estimate of the regression intercept was not statistically different from zero at 1% level of significance. As for the estimates of the risk factor coefficients it was found that at 1% level market beta estimates for 55% of the stocks were significantly different from zero, implied volatility beta estimates for 68% of the stocks were significantly different from zero, size beta estimates for 66% of the stocks significantly different from zero, and financial leverage beta estimates for 70% of the stocks significantly different from zero. Furthermore, in testing the first proposition of hypothesis five it was found that in the 855 regression results, the average adjusted R-square was 37.80% and median adjusted R-square was 41.60% with 94% of the stocks showing adjusted R-squares of above 50%. In testing of the second proposition of hypothesis five it was found that the intercept of the regression equation was not significantly different from zero, the estimate of all other coefficients were significantly different from zero and not significantly different from their expected values. Moreover, the adjusted R-square of the regression equation for the second proposition of the multifactor model was estimated to be 80.47%.

To sum up, in this research study it was found that despite the CAPM tenets the market rate of return is not the only risk factor determining the expected rates of return of individual stocks. Test of the hypotheses postulated in this study confirmed that the implied volatility of the overall market as a systematic risk factor and the companies' size and financial leverage as nonsystematic risk factors are important in determining stock's expected returns and investors should consider these factors in their investment decisions.

Conclusions

The findings of this research strongly confirm the idea that investment risk is the only reason why investors expect more rate of return from investing in a stock than from investing in the risk-free asset. This idea was inherent both in the standard single factor CAPM (hypothesis one) and the multifactor model (hypothesis five). This conclusion is backed by the finding in part one of hypothesis one that the estimate for the regression intercept for 99% of the stocks in the sample and by the finding in part one of hypothesis five that the estimate for the regression intercept for 98% of the stocks in the sample were not significantly different from zero at 1% level. If the intercept coefficient in regression Equation 41 under hypothesis one or the intercept coefficient in the regression Equation 44 under hypothesis five in chapter 4 is zero then the implication is that when there is no risk factor present in the economy expected rate of return of every stock would be the same as expected rate of return of the risk free-asset, the risk-free rate. In this respect the standard CAPM and the multifactor model developed by the author reach the same conclusion in light of empirical data.

However, less supportive for the CAPM's first proposition was the finding that out of the 855 stocks in the sample only for 56% of them the estimates for the slope coefficient, estimates of the beta, were significantly different from zero at 1% level. This means that the rates of return of 44% of the stocks in the sample were not sensitive to the rate of return of the market risk factor. Two possibilities might account for this finding. It could be that the S&P500 Stock Index is not the perfect proxy to represent the whole U.S. stock market and that is why some stock prices did not show correlation with the S&P 500 during the study period. Or as was discussed in the literature review chapter and proposed in hypotheses two through hypotheses five in chapter 3 and chapter 4 it could be that there are other risk factors, systematic or nonsystematic, that affect stock's rates of return. These conclusions are further substantiated by the finding reported in *Table 9* under testing of hypothesis one that for 88% of the stocks in the sample the adjusted R-square was below 20% and the median adjusted R-square was 6.60%. This very low explanatory power of the independent variable, the market risk factor, strongly suggests that there are some other independent variables missing in the model.

The findings in the test of the second proposition of CAPM confirm the conclusions derived from the first part. In regression Equation 43, in which average rates of return of cross section of stocks are regressed against their betas, it was found that the estimate for the intercept is significantly different from zero. This implies that for those stocks that have a beta of zero, those that do not respond to changes in the overall stock market, their rates of return are in excess of the risk-free rate. Given the previous finding that any excess rate of return is due to some risk factors, it becomes evident that there must be some other risk measures, besides the beta, that affect variations in stock's average rates of return. Also the finding that in regression Equation 43 the estimate for the coefficient of nonsystematic risk is not significantly different from zero confirm the idea advanced in hypotheses two through hypotheses five that systematic risk is not the only risk factor that affects stocks' expected rates of return. However, despite these shortcomings of the standard single CAPM, testing of the second part of hypothesis one supports the basic idea of CAPM that stocks' expected rates of return are positively related to their betas and that the slope of the line connecting the two is equal to expected excess rate of return of the market. The finding that the estimate for the coefficient of the beta in regression Equation 43 is not significantly different from monthly average excess rate of return of S&P 500 Stock Index over the study period strongly confirms the CAPM tenet, elaborated in the literature review section of this study and embodied in part two of hypothesis one, that the slope of the security market line (the SML) is positive and is equal to expected excess return of the market.

The conclusion that nonsystematic risk is important in determining stocks' rates of return was further reaffirmed by the findings in test of hypothesis two and test of hypothesis three. The finding in test of hypothesis two that realized rates of return of the stocks of small companies on average are higher than those of large companies confirms the conclusion that the nonsystematic risk factor, *size*, is a determining factor in stock's expected rates of return. Similarly, the finding in test of hypothesis three that realized rates of return of the stocks of high leverage companies on average are higher than those of low financial leverage companies confirms the conclusion that the nonsystematic risk factor, *financial leverage*, is a determining factor in stock's expected rates of return.

To compare and contrast the findings from testing the standard CAPM and the findings from testing of the multifactor model in terms of the explanatory power of the independent variables of each model, the adjusted R-square measures, rather than the R-square measures, were compared. The reason for doing this was that when new independent variables are added to a regression equation, the R-square measure automatically increases because addition of more independent variables leads to more regression sum of squares while total sum of squares remains the same. The adjusted R-square, however, is calculated after the regression sum of squares and the total sum of squares are divided by their respective degrees of freedom. Therefore, if the new added independent variables are not correlated with the dependent variable the adjusted R-square does not change very much. On the other hand, if the adjusted R-square does increase after inclusion of the new independent variables then addition of new independent variables in the regression equation is warranted and the increase in adjusted R-square is supporting evidence in favor of the explanatory power of the added variables.

The relationship between R-square and the adjusted R-square can be expressed through Equation 46 below (Aczel, 2002, pp. 511-514).

$$Adjusted\ R\text{-}square = 1\text{-}(1\text{-}R\text{-}sqaure)\frac{n-1}{n-(k+1)} \tag{46}$$

The findings in testing of the first proposition of the multifactor model, hypothesis five, confirm the effect and measure the extent of the additional systematic risk factor, implied volatility, and of the two nonsystematic risk factors, size and financial leverage, in determining individual stock's expected rate of return. As was reported in Table 28 in chapter 4, the solutions of regression Equation 44 for every individual stock in the sample showed that the average adjusted R-squares in the sample was 37.80%, the median adjusted R-square in the sample was 41.60% , and 94% of the stocks had adjusted R-square of above 50%. Comparing these results with similar findings in testing the first part of hypothesis one that the average adjusted R-square was 9.78%, the median adjusted r-square was 6.60%, and 89% of the stocks' adjusted R-squares were below 20%, demonstrate that the multifactor model provides a much better explanation for the factors that determine the rate of return of individual stocks. As was reported in Table 27 in chapter 4, this high explanatory power of the independent variables in explaining variations in individual stocks' rates of return in the multifactor model is further substantiated by the high percentage of the estimates of the risk factor coefficients, the betas, being significantly different from zero.

Results from test of the second proposition of the multifactor model in hypothesis five are very conclusive. As was reported in Table 30 and Table 31 in chapter 4, the four betas estimated for every stock in the sample proved to have strong effect on determining variation in stock's rates of return. The adjusted R-square in regression Equation 45 was 80.47%, quite higher than the adjusted R-square measure of 64.33% in Equation 43 of hypothesis one. This result shows that inclusion of the three extra independent variables, the implied volatility risk factor, the size risk factor, and the financial leverage risk factor provides a better explanation for variation in stock's expected rate of return than the standard single factor CAPM does. However, the finding that the estimate of the intercept in Equation 45 is not significantly different form zero suggests that there could still be some other risk factors which need to be incorporated into the multifactor model.

Implications for Social Change

The findings of this research have important implications for social change. The outcome of this study can change the way individual and institutional investors make investment decisions, can change the process that mutual funds select their portfolio managers and investment advisors, and can change capital budgeting and capital expenditure decisions of public corporations. These changes in turn could lead to significant changes in the resource allocation in the economy, in the economy's production capacity and production composition, and in the employment structure of the society.

The results of regression equations in hypothesis five can guide investors and portfolio managers to construct portfolios appropriate for desired levels of risks, assess the inherent risks of their portfolios, and manage portfolio risks to achieve returns objectives. Utilizing the beta estimates in regression Equation 45, investors and portfolio managers can assess various risks associated with any specific stock or with a portfolio of stocks and depict what if scenarios to predict the effect on their portfolios' return of anticipated or unanticipated changes in the risk factors. Investors and portfolio mangers can utilize the beta estimates of Equation 45 in their decisions to buy or sell individual stocks and thus affect the equilibrium stock prices in the stock market.

The outcome of this study can also help corporations, particularly large corporations, to estimate their cost of capital which is a pertinent factor when they want to evaluate investments in new projects or when deciding to rebalance their capital structures. Equation 45 of the multifactor model in chapter 4 can be used by corporations to estimate their cost of capital by employing the beta coefficients of their peer group company from those computed in this study. Moreover, the findings in hypothesis five and the beta estimates of Equation 45 can be used for the cost of capital calculation of public utilities; a problem that has always been an issue for the State Regulators in the US in determining public utilities rates.

The outcome of this study can assist investors and portfolio managers to identify undervalued or overvalued stocks and portfolios and make appropriate investment decisions. This can be done by first estimating the expected rate of return of a stock or portfolio of stocks, using the beta estimates of Equation 45, and then discounting future forecasted pay-offs of the investment at that expected rate of return to arrive at fair present value of the investment. This can also enable institutions to evaluate performance of their portfolio managers. Performance measurement is especially important in the mutual funds performance disclosures; and the outcome of this study can provide the ground for mutual funds to assess performance in connection with the amount of risk taken.

Recommendations for Action

The conclusions that there is an additional systematic risk factor, the market's implied volatility and two nonsystematic risk factors, companies' size and financial leverage affecting rates of return of stocks prompt specific actions to be implemented by various entities. Regulatory and supervisory authorities like the SEC and the NASD can provide educational materials for the investing public on their websites regarding these risk factors. They can also require financial advisors to disclose these risks when recommending specific investments to their clients. Institutional investors, like the mutual funds, that manage large amounts of investments including the nation's retirement accounts can direct their portfolio managers to incorporate these risk factors and their beta estimates in constructing appropriate portfolios. Additionally, large brokerage houses could instruct their investment banking departments, which value the initial public offering (IPO's) of corporations, to use the multifactor model proposed in this study in valuing the IPO's.

The results of this study can be disseminated through various means. The author intends to write a few papers on the basis of this research and hopefully have it published in peer reviewed journals. Excerpts of this study can be reworded in less specialized language and be published in the publicly read newspapers and magazines like the Wall Street Journal and the Investors' Business Daily that deal with investment literature. Finally, as a professional financial analyst the author will refer to and utilize the findings of this study when writing research reports for the client corporations.

Recommendations for Further Study

There are several areas that the author recommends and encourages further study in this field. First, as was noted both in hypothesis one and hypothesis five outcomes, the market beta for the individual stocks estimated form the time-series data were significantly different from zero for less than 60% of the stocks in the sample. One possibility for this outcome could be that the S&P500 Stock Index is not the best proxy for the overall US stock market. Therefore, conducting similar studies using other proxies for the overall market, possibly custom made, might lead to results stronger than those found in this study.

Another area for further research is to study the whole population of stocks trading in the U.S. stock market instead of studying a sample. This could be a tedious and time consuming process but the results will be totally conclusive leaving no room for sampling errors. Also, this study can be generalized to incorporate other stock markets in the developed countries. Or alternatively, the study can be done on a different sample in case the outcomes of the present study happen to be sample-specific.

Although in this study the R-square estimate computed in test of the second part of hypothesis five was much higher than the one obtained in test of the single factor CAPM, but still the adjusted R-square of about 80% leaves 20% of variations in the rates of return of the cross-section of stocks to remain unexplained. Thus, another area of further study is to identify other risk factors that affect stock’s rates of return and conduct hypotheses tests to verify their effects. One possibility that can be suggested here is to include the implied volatility of every individual stock as a nonsystematic risk factor in the multifactor model.

Concluding Statement

Economic growth and development and thus social prosperity can be achieved and maintained only through investments in the economy. Optimal investment decision making leads to optimal resource allocation and optimal economic growth. Optimal investment can be made only if relevant risk factors affecting the investment are identified and the degrees of their effectiveness are measured. This study was an attempt to identify and measure the risk factors involved in investing in the stock market. The take home message of this study for the average investor is to do adequate due diligence before committing funds into the stock market and or before selecting an investment advisor. This will benefit both the average investor and the whole economy.

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CURRICULUM VITAE

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Education

1972	M. Phil. (Post Master): University of Oxford, England, management studies, emphasis finance and econometrics.
1970	BA: University of Salford, England: Joint honors in mathematics, economics and statistics.
1967	GCE: University of London, England: Mathematics.

Teaching Experience

2001-Present	University of Phoenix, Southern California Campus: Adjunct faculty member, approved to teach and have taught online and on ground courses at BA and MBA level in corporate finance, managerial finance, statistics in business, and statistics in research.
1980-1992	College of Banking & Finance, Tehran; Teaching part- time courses in economics, finance and mathematics.
1973-1978	College of Computer Science & Planning, Tehran; Head of planning department devising curriculums for the department; teaching various courses in finance, mathematics & economics.
1972-1973	University of Oxford, England: Tutoring finance and mathematics.

Professional Experience

1993-Present	Financial analyst, writing research reports on US public companies. Independent financial advisor, managing portfolios of clients, quantitative research and analysis of companies and the stock market, selection, and risk analysis.
1978-1993	National Bank of Iran; Portfolio Manager and Investment Advisor: Evaluating, selecting, recommending and managing the Bank's Portfolio. Analyzing financial statements of companies going public and advising the Bank's board of directors. Representing the Bank on the board of various companies.

Professional Designations and Licenses

Charted Financial Analyst. (CFA Charter Holder)
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NASD licenses series 7, 24, 65, 86, and 87